

Radicals and Rational Exponents Made Clear

Grade 11 – Algebra 2 math · Lesson · Standards: N-RN.A.1, N-RN.A.2, A-REI.A.2

Quick answer: A rational exponent combines a power and a root: $a^{m/n}$ means the n -th root of a , raised to the m -th power. The denominator is the root index and the numerator is the power. Radical equations are solved by isolating the radical, raising both sides to the index, and then checking for extraneous solutions.

Standards: N-RN.A.1, N-RN.A.2, A-REI.A.2 · **Level:** Grade 11 Algebra 2

How are roots and rational exponents related?

An **n -th root** undoes raising to the n -th power. Rational exponents are a compact way to write roots and powers together.

Rational exponents

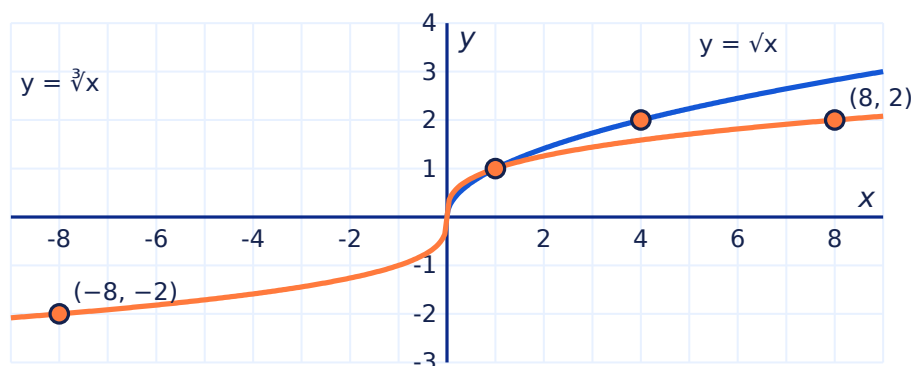
$$a^{1/n} = \sqrt[n]{a} \quad \text{and} \quad a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{(a^m)}$$

$$a^{(m/n)} = (\text{n-th root of } a)^m = \text{n-th root of } (a^m)$$

$$8^{(2/3)} = (\text{cube root of } 8)^2 = 2^2 = 4$$

Take the root first when the base is a perfect power; it keeps the numbers small.

Even roots of negative numbers are not real. Odd roots are: $\sqrt[3]{-8} = -2$. The graphs below show the difference.



The square root graph exists only for $x \geq 0$; the cube root graph covers all real numbers.

How do you simplify radical expressions?

Method: simplify a radical

1. Factor the radicand so one factor is a perfect n -th power.
2. Take the n -th root of that factor and move it outside.
3. Leave the remaining factor under the radical.

Worked example

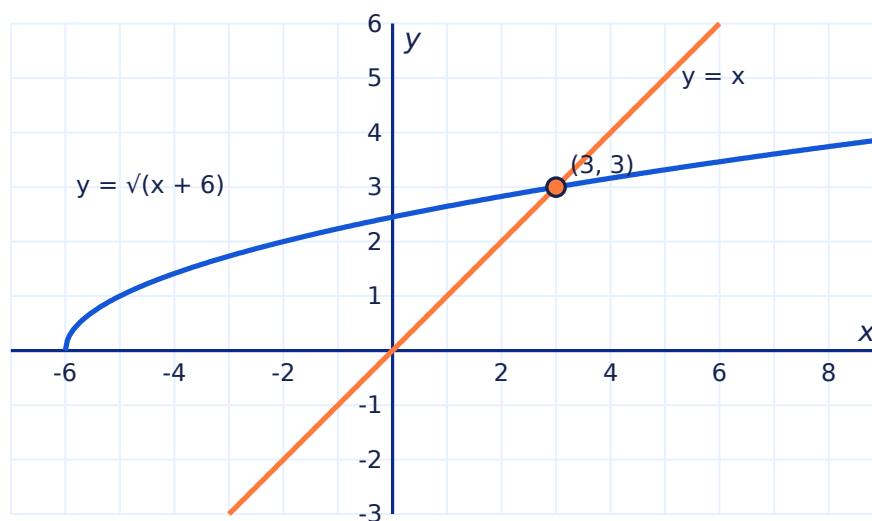
Simplify $\sqrt[3]{54}$. Since $54 = 27 \cdot 2$, $\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2} = \mathbf{3\sqrt[3]{2}}$. With exponents, $54^{1/3} = 27^{1/3} \cdot 2^{1/3}$.

How do you solve a radical equation?

Method: radical equations

1. Isolate the radical on one side.
2. Raise both sides to the power equal to the index.
3. Solve the resulting equation.
4. Substitute every answer into the original equation and discard extraneous ones.

Example: $\sqrt{x+6} = x$. Squaring gives $x+6 = x^2$, so $(x-3)(x+2) = 0$. Checking $x = 3$ gives $\sqrt{9} = 3$, which works. Checking $x = -2$ gives $\sqrt{4} = 2$, not -2 , so it is **extraneous**.



The graphs cross only at $x = 3$, so -2 is not a solution.

Common mistake

Squaring can create solutions that do not satisfy the original equation. Skipping the check is the most common error. Also, $(a+b)^{1/2}$ is not $a^{1/2} + b^{1/2}$.

Mathbot's tip

Read $a^{m/n}$ aloud as "root first, then power": the denominator is the root and the numerator is the power.

Key points to remember

- $a^{1/n}$ is the n -th root of a , and $a^{m/n} = ({}^n\sqrt{a})^m$.
- Odd roots of negative numbers are real; even roots of negative numbers are not.
- The usual exponent rules (add, subtract, multiply exponents) work for rational exponents.
- Simplify a radical by pulling out perfect n -th power factors.
- Squaring both sides can add extraneous solutions, so always check in the original equation.

Radicals and Rational Exponents Made Clear: frequently asked questions

What does $16^{3/4}$ mean?

It means the fourth root of 16 raised to the third power. The fourth root of 16 is 2, and 2 cubed is 8, so $16^{3/4} = 8$.

Why can an odd root of a negative number be real?

Because an odd power keeps the sign. For example, $(-2)^3 = -8$, so $\sqrt[3]{-8} = -2$. An even power is never negative, so even roots of negatives are not real.

What is an extraneous solution?

An extraneous solution appears while solving but fails in the original equation. Raising both sides of an equation to a power can create new solutions that the original equation does not have.

How do you rationalize a denominator?

Multiply the numerator and denominator by a quantity that removes the radical from the denominator. For $6/\sqrt{8}$ reduce first, then multiply by $\sqrt{2}/\sqrt{2}$; for a binomial such as $5/(\sqrt{7} - 2)$, multiply by the conjugate $\sqrt{7} + 2$.

When do you take the root first in a rational exponent?

Taking the root first is usually easier because the root of a perfect power is a small whole number. For $27^{2/3}$, take $\sqrt[3]{27} = 3$ first and then square it to get 9.