

Radicals and Rational Exponents Made Clear

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: N-RN.A.1, N-RN.A.2, A-REI.A.2

Each solution shows the conversion or check that matters most. When an equation is squared, the final answer is only the candidate that survives substitution into the original equation.

Standards: N-RN.A.1, N-RN.A.2, A-REI.A.2 · **Level:** Grade 11 Algebra 2

Problem 1 Evaluating roots

a) $(-5)^3 = -125$, so $\sqrt[3]{-125} = -5$.

b) $3^4 = 81$, so the fourth root is **3**.

c) $\sqrt{49} = 7$ and $\sqrt{64} = 8$, so $\frac{7}{8}$.

d) $2^5 = 32$, so **2**.

Problem 2 Evaluating rational exponents

a) $\sqrt{25} = 5$.

b) $(\sqrt[3]{8})^2 = 2^2 = 4$.

c) $(\sqrt[4]{16})^3 = 2^3 = 8$.

d) $(\sqrt{49})^3 = 7^3 = 343$.

Problem 3 Radical form to exponent form

a) $x^{1/2}$.

b) $x^{2/3}$.

c) $x^{3/4}$.

d) $y^{2/5}$.

Problem 4 Exponent form to radical form

- a) $\sqrt[3]{x}$.
- b) $\sqrt{(y^5)}$ or $(\sqrt{y})^5$.
- c) $\sqrt[4]{(7^3)}$ or $(\sqrt[4]{7})^3$.

Problem 5 Simplifying numeric radicals

- a) $162 = 81 \cdot 2$, so $9\sqrt{2}$.
- b) $40 = 8 \cdot 5$, so $2\sqrt[3]{5}$.
- c) $1,176 = 196 \cdot 6$, so $14\sqrt{6}$.

Problem 6 Reading a root graph

The graph passes through (9, 3) and (4, 2), so $9^{1/2} = 3$ and $4^{1/2} = 2$.

At $x = 12$ the curve is between 3 (at 9) and 4 (at 16). Since $3^2 = 9 < 12 < 16 = 4^2$, **$12^{1/2}$ lies between 3 and 4** (about 3.46).

Problem 7 Properties of rational exponents

- a) $1/2 + 1/3 = 5/6$: $x^{5/6}$.
- b) $(2/3)(3/4) = 1/2$: $x^{1/2}$.
- c) $3/4 - 1/4 = 1/2$: $x^{1/2}$.

Problem 8 Radicals with variables

- a) $18x^5 = 9x^4 \cdot 2x$, so $3x^2\sqrt{(2x)}$.
- b) $54a^4 = 27a^3 \cdot 2a$, so $3a\sqrt[3]{(2a)}$.

Problem 9 Combining like radicals

$$\sqrt{12} = 2\sqrt{3}, \sqrt{27} = 3\sqrt{3}, \sqrt{75} = 5\sqrt{3}.$$

$$10\sqrt{3} - 6\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}.$$

Problem 10 Multiplying radical expressions

a) $9 - 5 = 4$.

b) $4 \cdot 3 + 4\sqrt{3} + 1 = 13 + 4\sqrt{3}$.

Problem 11 Rationalizing denominators

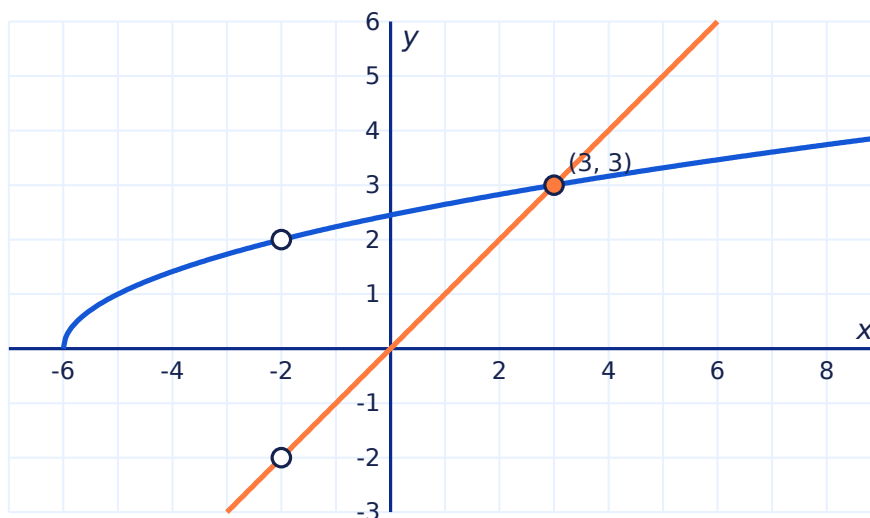
a) $\sqrt{8} = 2\sqrt{2}$, so $\frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

b) Multiply by the conjugate: $\frac{(5(\sqrt{7} + 2))}{((\sqrt{7})^2 - 4)} = \frac{(5(\sqrt{7} + 2))}{3} \cdot \frac{(5\sqrt{7} + 10)}{3}$

Problem 12 Solving with a graph

Square both sides: $x + 6 = x^2$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$. Candidates: 3 and -2 .

Check 3: $\sqrt{9} = 3$ ✓. Check -2 : $\sqrt{4} = 2 \neq -2$ ✗.



At $x = -2$ the curve and the line are different heights, so -2 is extraneous.

$x = 3$ (and -2 is extraneous)

Problem 13 Single radical equations

a) $3x - 5 = 16$, so $x = 7$. Check: $\sqrt{16} = 4$ ✓. **$x = 7$**

b) $2x + 1 = -27$, so $x = -14$. Check: $\sqrt[3]{(-27)} = -3$ ✓. **$x = -14$**

Problem 14 An extraneous solution

Square: $2x + 9 = x^2 + 6x + 9$, so $x^2 + 4x = 0$ and $x(x + 4) = 0$.

Check 0: $\sqrt{9} = 3$ and $0 + 3 = 3$ ✓. Check -4 : $\sqrt{1} = 1$ but $-4 + 3 = -1$ ✗.

$x = 0$ only

Problem 15 Isolating a radical first

Isolate the radical: $\sqrt{x + 14} = x + 2$. Square: $x + 14 = x^2 + 4x + 4$, so $x^2 + 3x - 10 = 0$, which factors as $(x + 5)(x - 2) = 0$.

Check 2: $\sqrt{16} = 4 = 2 + 2$ ✓. Check -5 : $\sqrt{9} = 3$ but $-5 + 2 = -3$ ✗.

$x = 2$

Problem 16 Two radicals

Isolate one radical: $\sqrt{x + 5} = 5 - \sqrt{x}$. Square: $x + 5 = 25 - 10\sqrt{x} + x$.

So $10\sqrt{x} = 20$, $\sqrt{x} = 2$, and $x = 4$. Check: $\sqrt{9} + \sqrt{4} = 3 + 2 = 5$ ✓.

$x = 4$

Problem 17 A rational exponent equation

Raise both sides to the power $3/2$: $|x - 4| = 9^{3/2} = 27$, so $x - 4 = 27$ or $x - 4 = -27$.

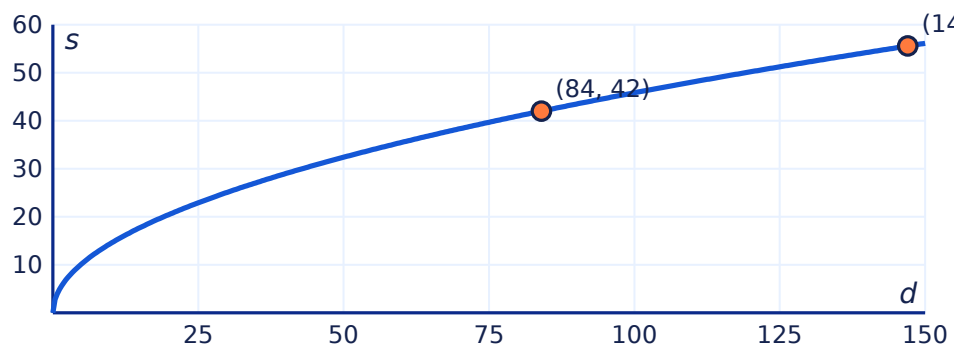
Check 31: $27^{2/3} = 9$ ✓. Check -23 : $(-27)^{2/3} = (\sqrt[3]{-27})^2 = 9$ ✓.

$x = 31$ or $x = -23$

Problem 18 Skid marks and speed

Set $\sqrt{21d} = 42$: $21d = 1,764$, so $d = 84$.

For $d = 147$: $s = \sqrt{21 \cdot 147} = \sqrt{3,087} = \sqrt{441 \cdot 7} = 21\sqrt{7} \approx 55.6$.



The two requested values.

84 feet; $21\sqrt{7}$ mph (about 55.6 mph)

Problem 19 Distance to the horizon

$12 = \sqrt{1.5h}$, so $144 = 1.5h$ and $h = 96$.

For $h = 6$: $d = \sqrt{9} = 3$.

The tower is 96 feet tall; the horizon is 3 miles away

Problem 20 Side of a cube

$s = 2,744^{1/3} = 14$, because $14^3 = 2,744$.

Surface area = $6s^2 = 6 \cdot 196 = 1,176$.

Side 14 inches; surface area 1,176 square inches