

Rational Expressions and Equations Step by Step

Grade 11 – Algebra 2 math · Lesson · Standards: A-APR.D.6, A-APR.D.7, A-REI.A.2

Quick answer: A rational expression is a quotient of polynomials. Simplify it by factoring and canceling common factors, add or subtract it with a least common denominator, and solve rational equations by multiplying by the LCD. Always reject solutions that make a denominator zero, because they are extraneous.

Standards: A-APR.D.6, A-APR.D.7, A-REI.A.2 · **Level:** Grade 11 Algebra 2

How do you simplify a rational expression?

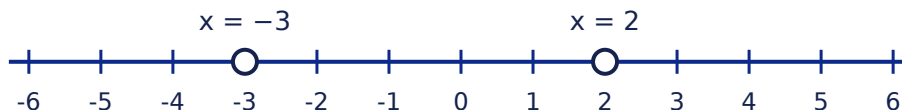
A **rational expression** is a quotient of two polynomials, such as $\frac{(x^2 - 4)}{(x^2 + x - 6)}$. To simplify it, factor the numerator and denominator, then cancel common *factors*, never common terms.

Method: simplify

1. Factor the numerator and the denominator completely.
2. List the values that make the original denominator 0.
3. Cancel each factor that appears in both.

Worked example

$$\frac{(x^2 - 4)}{(x^2 + x - 6)} = \frac{((x - 2)(x + 2))}{((x - 2)(x + 3))} = \frac{(x + 2)}{(x + 3)}, \text{ with } x \neq 2 \text{ and } x \neq -3.$$

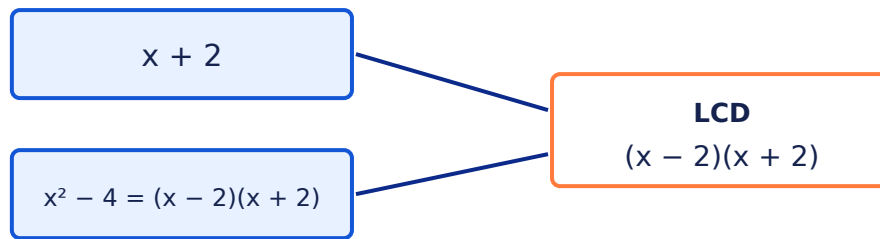


Excluded values for the example: the denominator is 0 at $x = 2$ and $x = -3$.

How do you multiply, divide, add, and subtract?

Operation	Rule	Example
Multiply	Multiply tops, multiply bottoms, cancel factors	$\frac{3}{x} \cdot \frac{x^2}{6} = \frac{x}{2}$
Divide	Multiply by the reciprocal of the second fraction	$\frac{x}{4} \div \frac{x}{8} = 2$
Add or subtract	Rewrite with a common denominator, then combine the numerators	$\frac{1}{x} + \frac{1}{3} = \frac{(3 + x)}{3x}$

For addition and subtraction, the **least common denominator (LCD)** is the product of every factor, using each factor the greatest number of times it appears in any one denominator.



Factor each denominator, then keep every factor once.

How do you solve a rational equation?

Method: rational equations

1. Factor all denominators and find the LCD.
2. Multiply both sides by the LCD to clear the fractions.
3. Solve the resulting equation.
4. Reject any answer that makes an original denominator 0.

Example: $\frac{6}{x} + \frac{3}{x+2} = 2$. The LCD is $x(x+2)$: $6(x+2) + 3x = 2x(x+2)$, so $2x^2 - 5x - 12 = 0$, which gives $x = 4$ or $x = -\frac{3}{2}$. Both values keep every denominator nonzero.

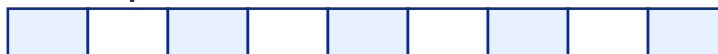
How do you solve work problems?

If a person finishes a job in t hours, the rate is $\frac{1}{t}$ of the job per hour. Rates add when people work together. Ana paints a fence in 6 hours and Ben in 9 hours; together $\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$ of the fence per hour, so the job takes $\frac{18}{5} = 3.6$ hours.

Ana: 1/6 per hour



Ben: 1/9 per hour



Together: 5/18 per hour



each bar = the whole job

Adding the hourly shares gives the combined rate.

Common mistake

Canceling terms instead of factors: $\frac{(x+3)}{(x+5)}$ does not simplify to $\frac{3}{5}$. Also, never skip the check for extraneous solutions.

Mathbot's tip

Write the excluded values first. They tell you in advance which answers will be extraneous.

Key points to remember

- Factor first, then cancel common factors, never common terms.
- A value that makes a denominator 0 is excluded from the domain.
- Add and subtract rational expressions with a common denominator; divide by multiplying by the reciprocal.
- To solve a rational equation, multiply by the LCD and check every answer in the original equation.
- In a work problem, rates of $1/t$ jobs per hour add.

Rational Expressions and Equations Step by Step: frequently asked questions

Why can't you cancel terms in a rational expression?

Only factors that multiply can be divided out. In $\frac{(x+3)}{(x+5)}$ the 3 and 5 are added to x , so they are terms. Cancelling them would change the value of the expression.

What is an excluded value?

An excluded value is a number that makes a denominator equal to 0, so the expression is undefined there. State excluded values before you simplify, because they stay excluded afterward.

How do you find a least common denominator?

Factor every denominator completely. The LCD contains each different factor the greatest number of times it occurs in any one denominator. For $x + 2$ and $x^2 - 4$, the LCD is $(x - 2)(x + 2)$.

What is an extraneous solution of a rational equation?

It is a number that solves the cleared equation but makes an original denominator 0. Because division by 0 is undefined, such a number cannot be a solution and must be discarded.

How do you set up a work problem?

Write each rate as 1 divided by the time to finish alone. Add the rates for people working together and set the sum equal to 1 divided by the combined time.