

Rational Expressions and Equations Step by Step

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: A-APR.D.6, A-APR.D.7, A-REI.A.2

Each solution factors first, names the common denominator, and checks the final answers against the excluded values. Compare each step with the method in the lesson.

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Problem 1 Simplifying monomial fractions

Divide the numbers by 6 and cancel powers: $12/18 = 2/3$, $x^3/x = x^2$, $y/y^2 = 1/y$.

$$\frac{2x^2}{3y}$$

Problem 2 Excluded values

a) $x^2 - 9 = (x - 3)(x + 3)$, so $x \neq 3$ and $x \neq -3$.

b) $2x - 7 = 0$ at $x = \frac{7}{2}$.

c) $x^2 + x - 12 = (x + 4)(x - 3)$, so $x \neq -4$ and $x \neq 3$.

(a) ± 3 ; (b) $\frac{7}{2}$; (c) -4 and 3

Problem 3 Simplifying by factoring

Numerator: $(x - 5)(x + 5)$. Denominator: $(x - 5)(x + 2)$.

Cancel $x - 5$: $\frac{(x + 5)}{(x + 2)}$, with $x \neq 5$ and $x \neq -2$.

Problem 4 Multiplying rational expressions

The factor $x + 5$ cancels, and $3x^2/(9x) = x/3$.

$\frac{x}{3}$, with $x \neq -5$ and $x \neq 0$.

Problem 5 Excluded values on a number line

$x^2 - x - 20 = (x - 5)(x + 4) = 0$ at $x = 5$ and $x = -4$.



Excluded values -4 and 5 .

Domain: all real numbers except -4 and 5

Problem 6 Dividing rational expressions

Multiply by the reciprocal: $\frac{(x-2)(x+2)}{(x+3)} \cdot \frac{(x+3)}{(x+2)}$.

Cancel $x + 2$ and $x + 3$ to get $x(x - 2)$, with $x \neq -3, -2, 0$.

$x(x - 2)$

Problem 7 Like denominators

a) $\frac{(3x+1+2x-9)}{(x-4)} = \frac{(5x-8)}{(x-4)}$.

b) Distribute the minus: $\frac{(7x+2-x+10)}{(x+6)} = \frac{(6x+12)}{(x+6)} = \frac{(6(x+2))}{(x+6)}$.

Problem 8 Unlike denominators

The LCD is $(x + 5)(x - 3)$.

$$\frac{(7(x-3)+2(x+5))}{((x+5)(x-3))} = \frac{(7x-21+2x+10)}{((x+5)(x-3))}.$$

$$\frac{(9x-11)}{((x+5)(x-3))}$$

Problem 9 Factoring a denominator first

$x^2 - 9 = (x - 3)(x + 3)$, so the LCD is $(x - 3)(x + 3)$.

$$\frac{(5-2(x+3))}{((x-3)(x+3))} = \frac{(5-2x-6)}{((x-3)(x+3))}.$$

$$\frac{(-2x-1)}{((x-3)(x+3))}$$

Problem 10 Adding with a quadratic denominator

$x^2 - 5x + 6 = (x - 2)(x - 3)$, so the second fraction needs the factor $x - 3$.

$$\frac{(x + 4(x - 3))}{((x - 2)(x - 3))} = \frac{(x + 4x - 12)}{((x - 2)(x - 3))}$$

$$\frac{(5x - 12)}{((x - 2)(x - 3))}$$

Problem 11 A first rational equation

Multiply every term by x : $8 - 3x = 2$, so $3x = 6$ and $x = 2$.

Check: $8/2 - 3 = 1$ and $2/2 = 1$ ✓.

$$x = 2$$

Problem 12 No solution

Multiply by $x - 5$: $x + 2(x - 5) = 5$, so $3x = 15$ and $x = 5$.

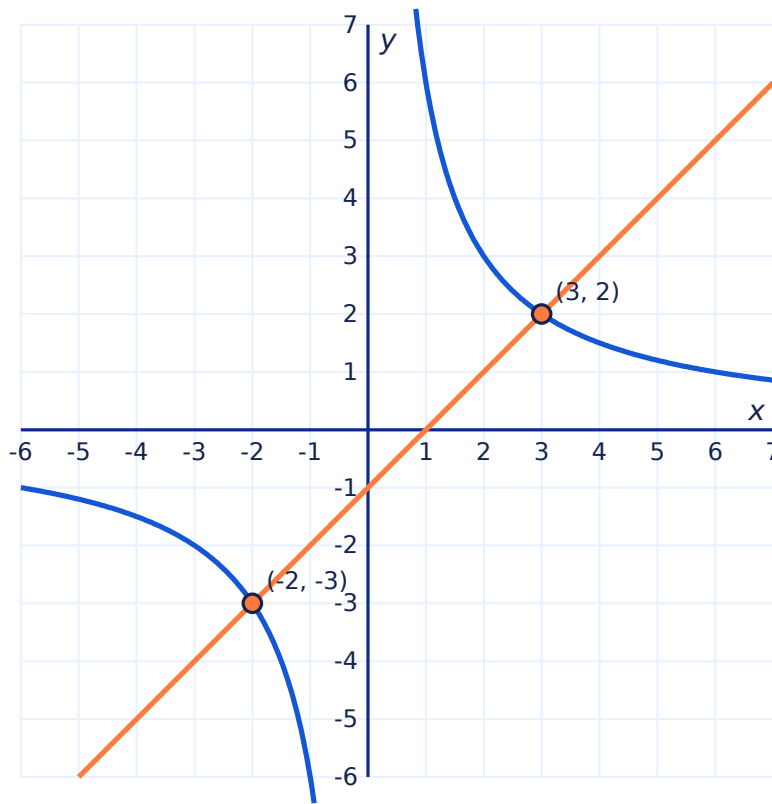
But $x = 5$ makes the denominator 0, so it is extraneous.

No solution

Problem 13 Solving with a graph

The graphs cross at $(3, 2)$ and $(-2, -3)$.

Algebra: multiply by x : $6 = x^2 - x$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$.



Both solutions are valid.

$x = 3$ and $x = -2$

Problem 14 Two denominators and a quadratic

Multiply by $(x - 1)(x + 4)$: $3(x + 4) + 2(x - 1) = (x - 1)(x + 4)$.

$5x + 10 = x^2 + 3x - 4$, so $x^2 - 2x - 14 = 0$.

$$x = \frac{(2 \pm \sqrt{4 + 56})}{2} = \frac{(2 \pm 2\sqrt{15})}{2} = 1 \pm \sqrt{15}.$$

Neither value equals 1 or -4 , so both are valid.

$x = 1 + \sqrt{15}$ or $x = 1 - \sqrt{15}$

Problem 15 Rejecting an extraneous solution

The LCD is $(x - 2)(x + 2)$. Multiply: $3(x + 2) - 12 = x^2 - 4$, so $3x - 6 = x^2 - 4$ and $x^2 - 3x + 2 = 0$.

$(x - 1)(x - 2) = 0$, so $x = 1$ or $x = 2$.

$x = 2$ makes a denominator 0, so reject it. Check $x = 1$: $-3 + 4 = 1$ ✓.

$x = 1$

Problem 16 Painting a fence together

Ana: $\frac{1}{6}$ per hour. Ben: $\frac{1}{9}$ per hour. Together: $\frac{1}{6} + \frac{1}{9} = \frac{3}{18} + \frac{2}{18} = \frac{5}{18}$ per hour.

Time = $1 \div \frac{5}{18} = \frac{18}{5} = 3.6$ hours.

Ana: $\frac{1}{6}$ per hour



Ben: $\frac{1}{9}$ per hour



Together: $\frac{5}{18}$ per hour



each bar = the whole job

Together, 3.6 hours fill the whole job.

3.6 hours (3 hours 36 minutes)

Problem 17 Two hoses filling a pond

Let t be the hours for the faster hose; the slower takes $t + 5$. Rates: $\frac{1}{t} + \frac{1}{t+5} = \frac{1}{6}$.

Multiply by $6t(t + 5)$: $6(t + 5) + 6t = t(t + 5)$, so $12t + 30 = t^2 + 5t$.

$t^2 - 7t - 30 = 0$, which factors as $(t - 10)(t + 3) = 0$. The time must be positive, so $t = 10$.

Check: $\frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6} \checkmark$.

Faster hose 10 hours; slower hose 15 hours

Problem 18 Paddling against a current

Let s be the still-water speed. Upstream speed $s - 3$, downstream $s + 3$. Total time:

$$\frac{12}{s-3} + \frac{12}{s+3} = 3.$$

Multiply by $(s - 3)(s + 3)$: $12(s + 3) + 12(s - 3) = 3(s^2 - 9)$, so $24s = 3s^2 - 27$ and $s^2 - 8s - 9 = 0$.

$(s - 9)(s + 1) = 0$. A speed must be positive, so $s = 9$.

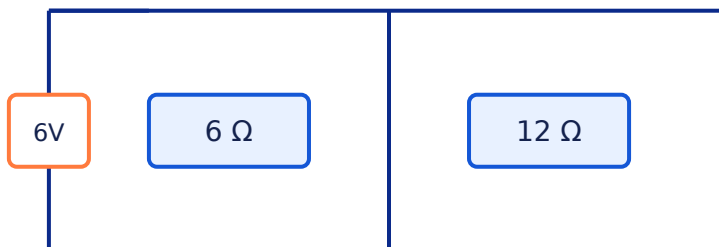
Check: $\frac{12}{6} + \frac{12}{12} = 2 + 1 = 3 \checkmark$.

9 mph

Problem 19 Resistors in parallel

$\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$. Multiply by $4x(x+6)$: $4(x+6) + 4x = x(x+6)$.

$8x + 24 = x^2 + 6x$, so $x^2 - 2x - 24 = 0$ and $(x-6)(x+4) = 0$. A resistance is positive, so $x = 6$.



two resistors in parallel
Resistances 6 and 12 ohms.

Check: $\frac{1}{6} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ ✓.

6 ohms and 12 ohms

Problem 20 A multi-step quotient

Rewrite as a product: $\frac{((x-3)(x+3))}{(x(x+2))} \cdot \frac{((x-2)(x+2))}{(x+3)}$.

Cancel $x+3$ and $x+2$: $\frac{((x-3)(x-2))}{x}$.

Excluded: $x = 0$ and $x = -2$ (original denominator), $x = -3$ (divisor is 0), and $x = \pm 2$ (from $x^2 - 4$).

$\frac{((x-3)(x-2))}{x}$, with $x \neq 0, -2, -3, 2$