

Rational Functions: Asymptotes, Holes, and Graphs

Grade 11 – Algebra 2 math · Test Answer Key · Standards: F-IF.C.7d, F-IF.B.4

Each solution names the feature and shows the factoring or substitution behind it. Award partial credit for correct reasoning with a small slip, and check that asymptotes are written as equations, not just numbers.

Standards: F-IF.C.7d, F-IF.B.4 · **Level:** Grade 11 Algebra 2

Question 1 Domain and asymptotes (2 points)

The denominator is 0 at $x = -6$. Equal degrees give $y = 4/1 = 4$.

Domain: all reals except -6 ; vertical asymptote $x = -6$; horizontal asymptote $y = 4$

Scoring: 1 point for the domain and vertical asymptote, 1 point for the horizontal asymptote.

Question 2 Transformations (2 points)

The factor -7 stretches vertically by 7 and reflects across the x-axis; $x - 5$ shifts right 5; -2 shifts down 2.

Reflect, stretch by 7, right 5, down 2

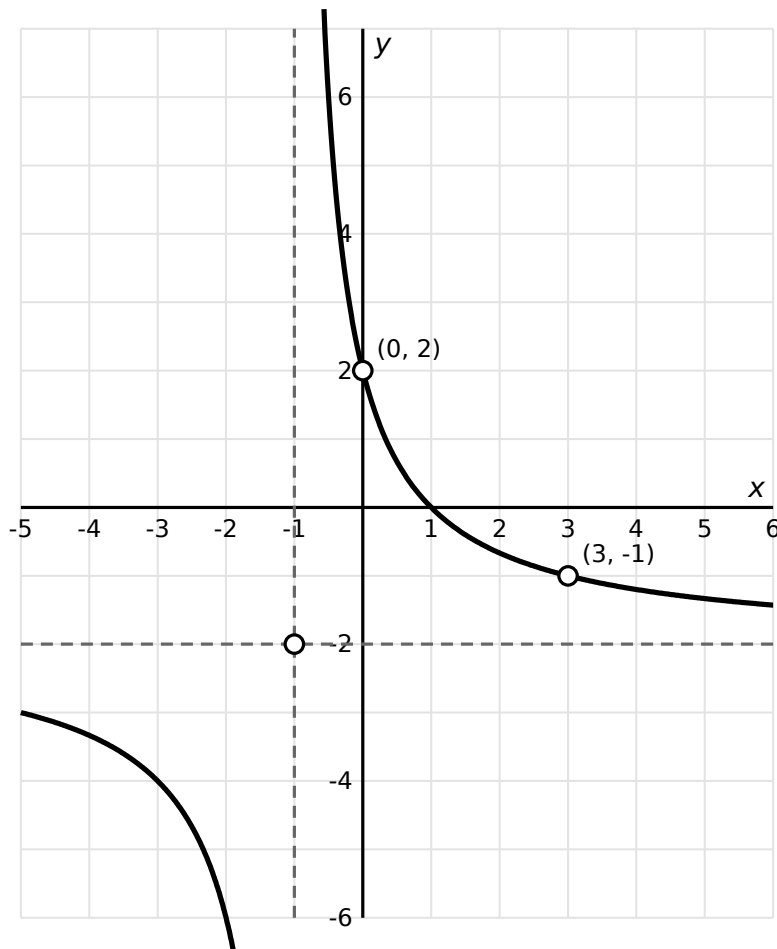
Scoring: 1 point for the reflection and stretch, 1 point for the two shifts.

Question 3 From graph to equation (3 points)

a) The dashed lines are **$x = -1$ and $y = -2$** .

b) $y = \frac{a}{(x+1)} - 2$. At $(0, 2)$: $2 = a - 2$, so $a = 4$. Check $(3, -1)$: $4/4 - 2 = -1$ ✓.

$$y = \frac{4}{(x+1)} - 2$$



The graph and asymptotes.

Scoring: 1 point for the asymptotes, 1 for a , 1 for the equation.

Question 4 Hole, asymptotes (3 points)

Factor: $\frac{(x-5)(x+4)}{((x-5)(x+5))} = \frac{(x+4)}{(x+5)}$ for $x \neq 5$.

Hole at $x = 5$ with height $9/10$. Vertical asymptote $x = -5$. Horizontal asymptote $y = 1$.

Hole $(5, \frac{9}{10})$; $x = -5$; $y = 1$

Scoring: 1 point each feature.

Question 5 Inverse variation (3 points)

a) $k = 7 \cdot 8 = 56$, so $y = \frac{56}{x}$.

b) $y = 56/14 = 4$.

c) $2 = 56/x$, so $x = 28$.

Scoring: 1 point per part.

Question 6 Average cost (3 points)

a) $A(150) = 5 + 900/150 = 5 + 6 = \text{\$11}.$

b) $5 + 900/n = 8$, so $900/n = 3$ and $n = \text{300}.$

c) **A = 5**: as more items are printed, the average cost approaches the \$5 cost per item.

Scoring: 1 point per part.

Question 7 Rewrite and identify (2 points)

$4x - 7 = 4(x - 3) + 5$, so $\frac{(4x - 7)}{(x - 3)} = 4 + \frac{5}{(x - 3)}.$

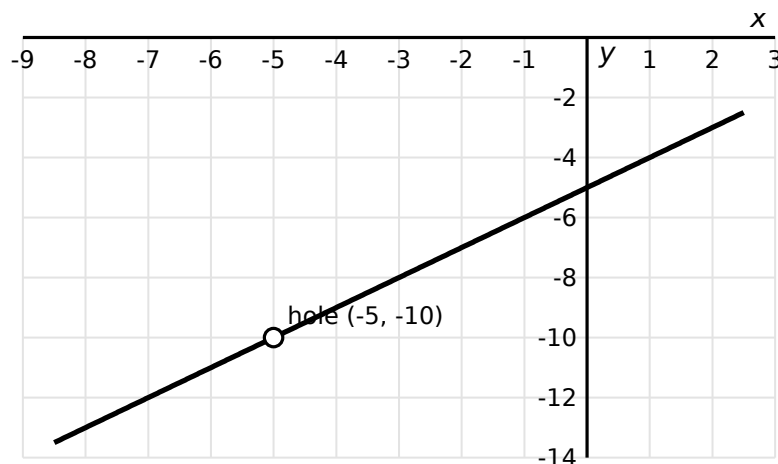
$y = \frac{5}{(x - 3)} + 4$; asymptotes $x = 3$ and $y = 4$

Scoring: 1 point for the rewrite, 1 point for the asymptotes.

Question 8 A hole on a graph (2 points)

a) $\frac{(x - 5)(x + 5)}{(x + 5)} = x - 5$, so the hole is at $x = -5$ with height -10 : **$(-5, -10)$** .

b) **All real numbers except -5 .**



The hole.

Scoring: 1 point per part.