

Rational Functions: Asymptotes, Holes, and Graphs

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: F-IF.C.7d, F-IF.B.4

Each solution names the key feature and the algebra that exposes it. Compare each sketch with the asymptotes and points given, and check holes by canceling factors.

Standards: F-IF.C.7d, F-IF.B.4 · **Level:** Grade 11 Algebra 2

Problem 1 Finding the domain

- a) $x - 3 = 0$ at $x = 3$: **all real numbers except 3.**
- b) $x^2 - 16 = (x - 4)(x + 4)$: **all real numbers except 4 and -4.**
- c) $2x + 5 = 0$ at $x = -\frac{5}{2}$: **all real numbers except $-\frac{5}{2}$.**

Problem 2 Asymptotes from the equation

- a) The form $a/(x - h) + k$ has $h = -4$ and $k = -3$: **$x = -4$ and $y = -3$.**
- b) $h = 5$ and $k = 6$: **$x = 5$ and $y = 6$.**

Problem 3 Describing transformations

Replacing x by $x + 3$ shifts the graph left 3. Subtracting 5 shifts it down 5.

Left 3 and down 5; the asymptotes move to $x = -3$ and $y = -5$.

Problem 4 Reading a graph

The branches approach the dashed lines $x = 1$ and $y = 2$.

Asymptotes $x = 1$ and $y = 2$; domain: all reals except 1; range: all reals except 2

Problem 5 Inverse variation constant

$$k = xy = 3 \cdot 12 = 36, \text{ so } y = \frac{36}{x}.$$

When $x = 9$: $y = 36/9 = 4$.

$$y = \frac{36}{x}; y = 4$$

Problem 6 A hole in the graph

$\frac{(x-6)(x+6)}{(x-6)} = x+6$ for $x \neq 6$. At $x = 6$ the simplified expression gives 12.

Hole at (6, 12)

Problem 7 A hole and an asymptote

The factor $x+2$ cancels, leaving $\frac{1}{(x-5)}$. The hole is at $x = -2$, with height $1/(-2-5) = -\frac{1}{7}$.

Hole at $(-2, -\frac{1}{7})$; vertical asymptote $x = 5$

Problem 8 Horizontal asymptotes by degree

a) Equal degrees: ratio of leading coefficients $3/1$: **$y = 3$** .

b) Numerator degree 1 < denominator degree 2: **$y = 0$** .

c) Numerator degree 3 > denominator degree 2: **none**.

Problem 9 Writing an equation from features

$h = -3$ and $k = 2$, so $y = \frac{a}{(x+3)} + 2$. At $(0, 3)$: $3 = a/3 + 2$, so $a = 3$.

$$y = \frac{3}{(x+3)} + 2$$

Problem 10 Intercepts and asymptotes

x-intercept: $2x - 6 = 0$ gives $x = 3$. y-intercept: $y = -6/1 = -6$.

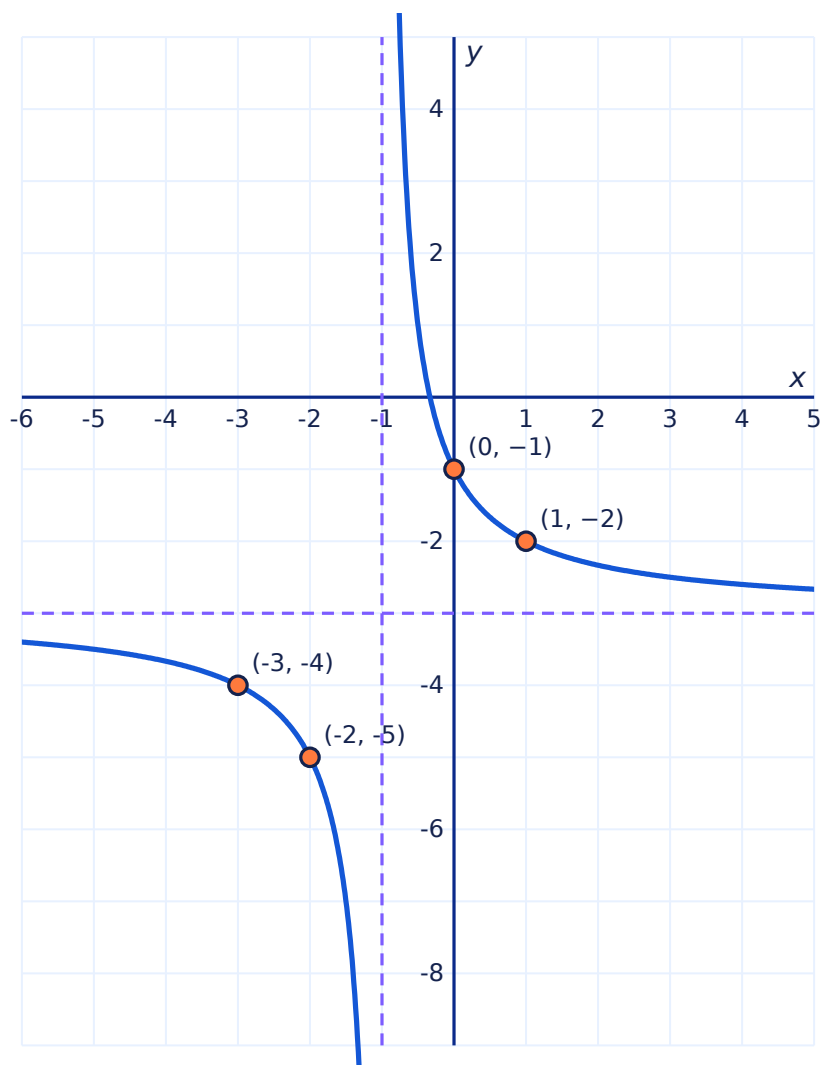
Vertical asymptote where $x + 1 = 0$. Equal degrees: horizontal asymptote $y = 2/1 = 2$.

(3, 0); (0, -6); $x = -1$; $y = 2$

Problem 11 Sketching a rational function

Asymptotes: $x = -1$ and $y = -3$. Since $a = 2 > 0$, the branches are above-right and below-left of the crossing point $(-1, -3)$.

Points: $x = 0$ gives -1 ; $x = 1$ gives -2 ; $x = -2$ gives -5 ; $x = -3$ gives -4 .



The completed sketch.

Asymptotes $x = -1$ and $y = -3$

Problem 12 Is it inverse variation?

Products: $4 \cdot 15 = 60$, $5 \cdot 12 = 60$, $6 \cdot 10 = 60$, $10 \cdot 6 = 60$. The product is constant, so y varies inversely with x .

$y = \frac{60}{x}$. At $x = 15$: $y = 4$.

$y = \frac{60}{x}$; $y = 4$

Problem 13 Rewriting in standard form

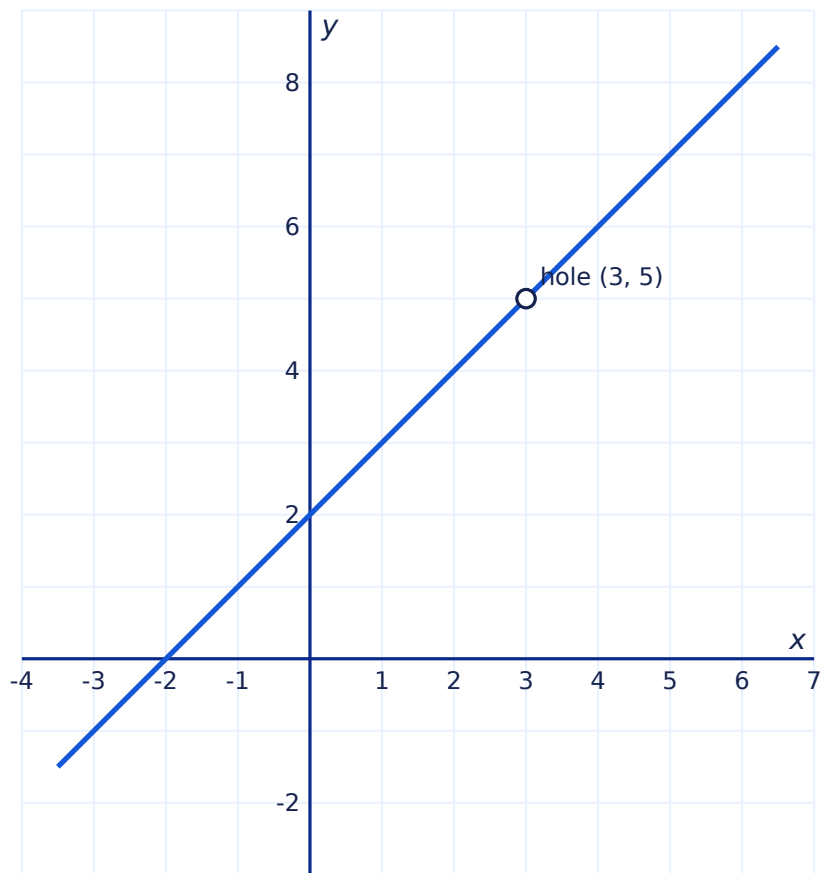
$3x + 5 = 3(x + 2) - 1$, so $\frac{(3x + 5)}{(x + 2)} = 3 - \frac{1}{(x + 2)}$.

So $y = -\frac{1}{(x+2)} + 3$.

$x = -2$ and $y = 3$

Problem 14 Domain and hole from a graph

Factor: $\frac{(x-3)(x+2)}{(x-3)} = x+2$ for $x \neq 3$. The hole is at $x = 3$, with height $3 + 2 = 5$.



The hole is at (3, 5).

Hole (3, 5); domain all reals except 3; line $y = x + 2$

Problem 15 Solving for an input

$f(x) = 5$: $6/(x-1) = 3$, so $x-1 = 2$ and $x = 3$.

$f(x) = 0$: $6/(x-1) = -2$, so $x-1 = -3$ and $x = -2$.

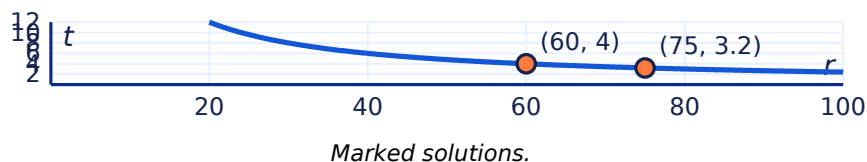
$x = 3$; $x = -2$

Problem 16 A road trip

At 60 mph: $t = 240/60 = 4$ hours.

For $t = 3.2$: $r = 240/3.2 = 75$ mph.

As r grows, t gets closer to 0 but never reaches it, so the horizontal asymptote is $t = 0$.



4 hours; 75 mph; the time approaches 0

Problem 17 Diluting a solution

$\frac{2}{10+x} = 0.1$, so $10 + x = 20$ and $x = 10$.

Domain: $x \geq 0$. Horizontal asymptote: $c = 0$, since the numerator has the smaller degree.

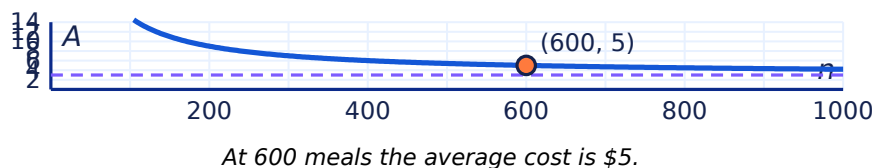
10 liters of water; asymptote $c = 0$: the acid fraction approaches 0 but never reaches it

Problem 18 Average cost of a food truck

$$(3n + 1,200)/n = 3 + 1,200/n.$$

Set $3 + 1,200/n = 5$: $1,200/n = 2$, so $n = 600$.

As n grows, $1,200/n$ approaches 0, so A approaches 3.



$A(n) = 3 + 1,200/n$; $n = 600$; horizontal asymptote $A = 3$

Problem 19 All the features of one function

Factor: $\frac{(x-4)(x+3)}{(x-4)(x+2)}$. The factor $x - 4$ cancels, leaving $\frac{(x+3)}{(x+2)}$.

Hole at $x = 4$: height $7/6$. Vertical asymptote: $x = -2$. Horizontal asymptote: equal degrees, $y = 1$.

x-intercept: $x + 3 = 0$ gives $x = -3$. y-intercept: $f(0) = -12/-8 = \frac{3}{2}$.

Hole $(4, \frac{7}{6})$; $x = -2$; $y = 1$; $(-3, 0)$; $(0, \frac{3}{2})$

Problem 20 Designing a rational function

The denominator needs the factors $(x - 4)$ for the asymptote and $(x + 3)$ for the hole, and the numerator must also contain $(x + 3)$. For the x-intercept 7 include $(x - 7)$.

Equal degrees with a ratio of leading coefficients 5 requires a factor 5 in the numerator.

$f(x) = \frac{5(x - 7)(x + 3)}{(x - 4)(x + 3)}$. The simplified form is $\frac{5(x - 7)}{(x - 4)}$; at $x = -3$ it equals $5(-10)/(-7) = \frac{50}{7}$.

$f(x) = \frac{5(x - 7)(x + 3)}{(x - 4)(x + 3)}$; hole at $(-3, \frac{50}{7})$