

Rational Functions: Asymptotes, Holes, and Graphs

Grade 11 – Algebra 2 math · Worksheet · Standards: F-IF.C.7d, F-IF.B.4

Name: _____ Date: _____ Class: _____

These problems explore rational functions through domain, asymptotes, holes, intercepts, and graphs of $y = a/(x - h) + k$. They match Common Core standards F-IF.C.7d and F-IF.B.4.

The set progresses from reading features off equations and graphs to rewriting expressions, sketching, and interpreting inverse variation and average cost models. Factor and cancel first to separate holes from asymptotes.

Standards: F-IF.C.7d, F-IF.B.4 · **Level:** Grade 11 Algebra 2

Problem 1 Finding the domain (easy)

State the domain of each function.

- a. $f(x) = \frac{5}{(x-3)}$
- b. $g(x) = \frac{(x+1)}{(x^2-16)}$
- c. $h(x) = \frac{7}{(2x+5)}$

Problem 2 Asymptotes from the equation (easy)

Give the equations of the vertical and horizontal asymptotes.

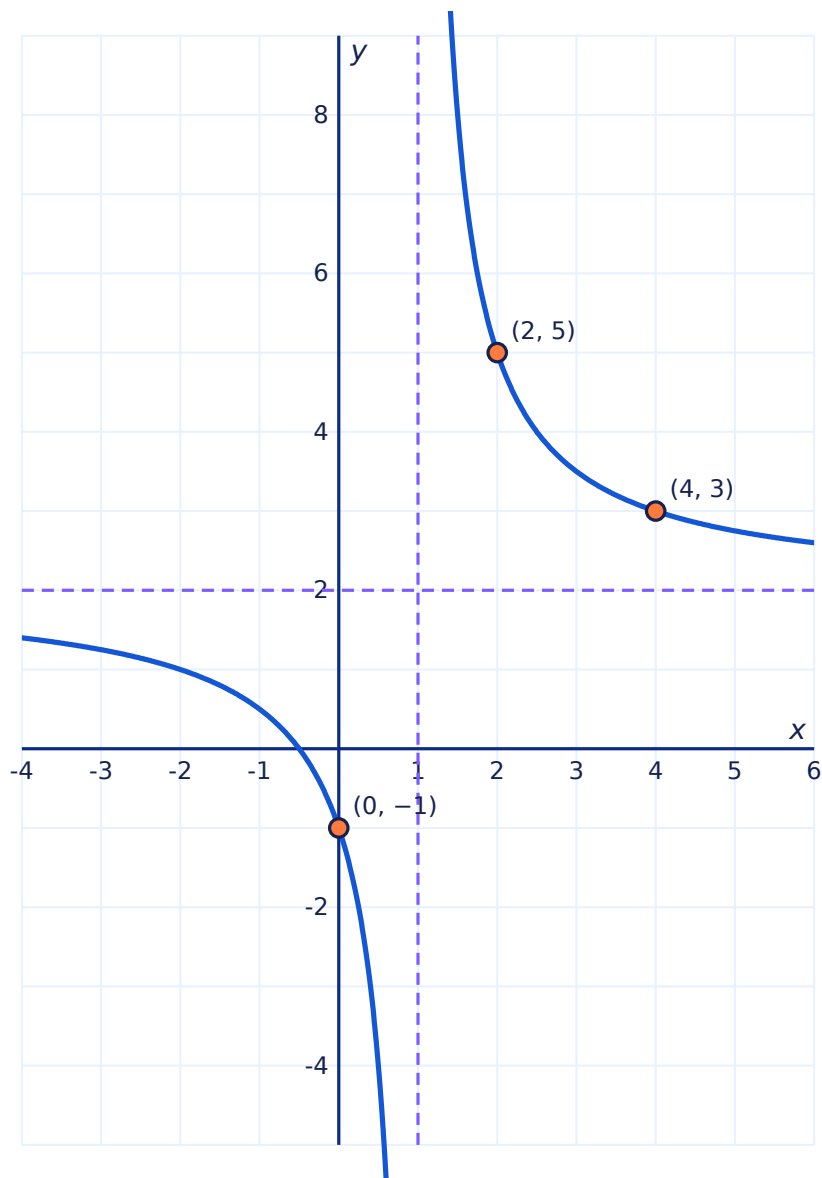
- a. $y = \frac{2}{(x+4)} - 3$
- b. $y = -\frac{1}{(x-5)} + 6$

Problem 3 Describing transformations (easy)

Describe how the graph of $y = \frac{1}{(x+3)} - 5$ is obtained from the graph of $y = \frac{1}{x}$.

Problem 4 Reading a graph (easy)

The graph of a rational function is shown. State the equations of its asymptotes, its domain, and its range.



A rational function graph.

Problem 5 Inverse variation constant (easy)

The quantity y varies inversely with x , and $y = 12$ when $x = 3$. Write the equation and find y when $x = 9$.

Problem 6 A hole in the graph (medium)

Simplify $f(x) = \frac{(x^2 - 36)}{(x - 6)}$ and find the coordinates of the hole.

Problem 7 A hole and an asymptote (medium)

For $f(x) = \frac{(x + 2)}{((x + 2)(x - 5))}$, find the hole and the vertical asymptote.

Problem 8 Horizontal asymptotes by degree (medium)

Find the horizontal asymptote of each function, or state that there is none.

a. $f(x) = \frac{(3x^2 + 1)}{(x^2 - 4)}$

b. $g(x) = \frac{(5x + 2)}{(x^2 + 1)}$

c. $h(x) = \frac{x^3}{(2x^2 + 1)}$

Problem 9 Writing an equation from features (medium)

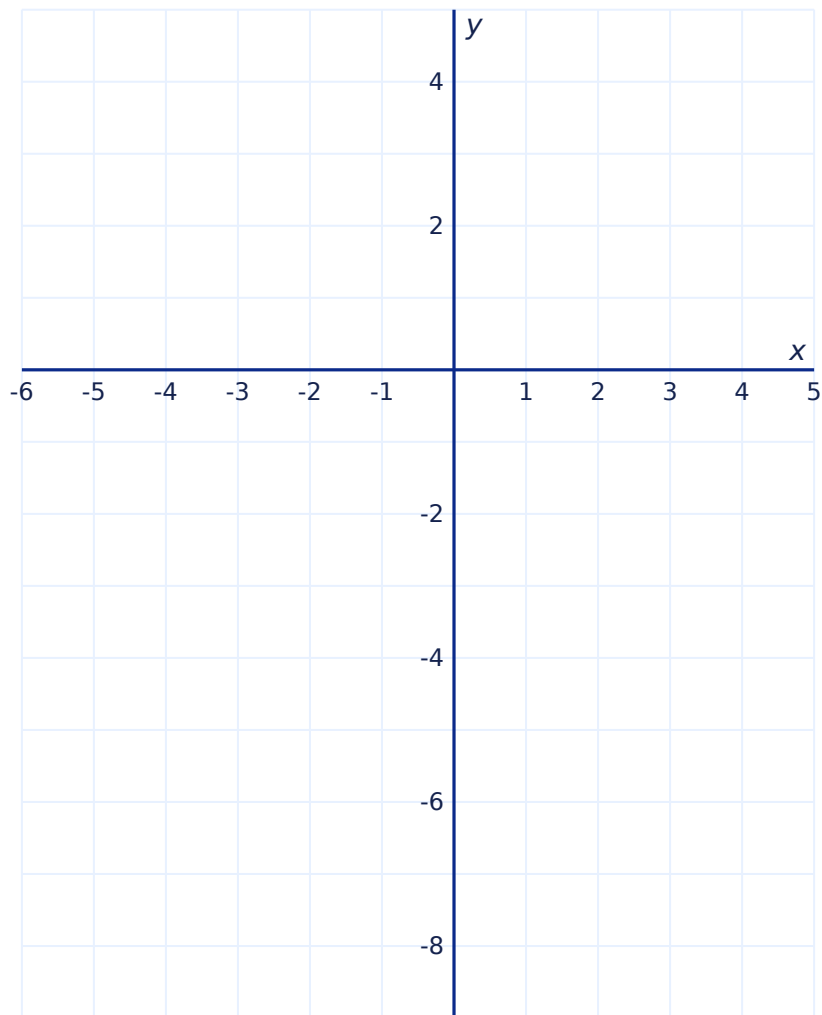
A rational function $y = \frac{a}{(x - h)} + k$ has vertical asymptote $x = -3$, horizontal asymptote $y = 2$, and passes through $(0, 3)$. Find a and write the equation.

Problem 10 Intercepts and asymptotes (medium)

For $y = \frac{(2x - 6)}{(x + 1)}$, find the x-intercept, the y-intercept, and both asymptotes.

Problem 11 Sketching a rational function (medium)

Sketch the graph of $y = \frac{2}{(x + 1)} - 3$. Draw both asymptotes and plot at least four points.



Grid for the sketch.

Problem 12 Is it inverse variation? (medium)

The table shows pairs from a situation. Show that y varies inversely with x , write the equation, and find y when $x = 15$.

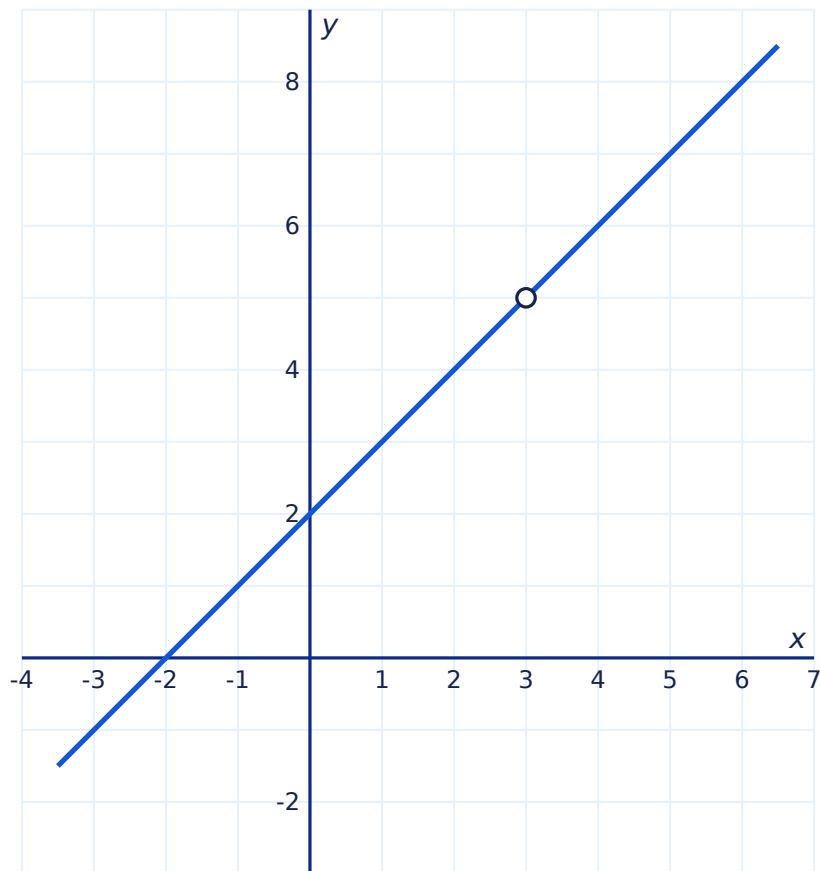
x	4	5	6	10
y	15	12	10	6

Problem 13 Rewriting in standard form (medium)

Divide to rewrite $y = \frac{(3x + 5)}{(x + 2)}$ in the form $y = \frac{a}{(x - h)} + k$, and state the asymptotes.

Problem 14 Domain and hole from a graph (medium)

The graph is of $f(x) = \frac{(x^2 - x - 6)}{(x - 3)}$. Identify the hole, give the domain, and write the equation of the line that the graph follows.



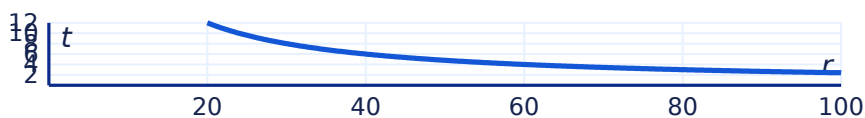
A line with a hole.

Problem 15 Solving for an input (hard)

For $f(x) = \frac{6}{x-1} + 2$, find the value of x for which $f(x) = 5$, and the value of x for which $f(x) = 0$.

Problem 16 A road trip (hard)

A road trip covers 240 miles, so the time in hours at a steady speed of r mph is $t = \frac{240}{r}$. Find the time at 60 mph, find the speed needed to finish in 3.2 hours, and describe what the graph does as r grows.



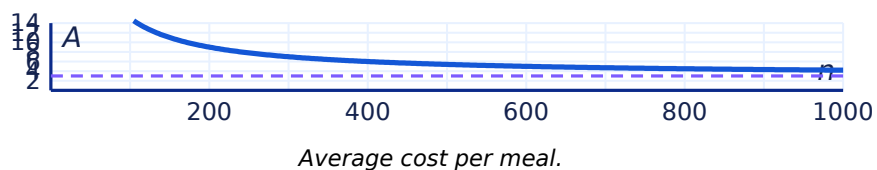
Time against speed.

Problem 17 Diluting a solution (hard)

A beaker holds 10 liters of liquid that contains 2 liters of acid. After x liters of water are added, the fraction of acid is $c(x) = \frac{2}{10+x}$. How much water must be added so that the fraction is 0.1? Give the domain in this context and the horizontal asymptote, and interpret it.

Problem 18 Average cost of a food truck (hard)

A food truck has a \$1,200 start-up cost and spends \$3 on ingredients per meal. The average cost per meal after n meals is $A(n) = \frac{(3n + 1,200)}{n}$. Rewrite $A(n)$ in the form $3 + \frac{k}{n}$, find n when the average cost is \$5, and name the horizontal asymptote.

**Problem 19 All the features of one function** (hard)

For $f(x) = \frac{(x^2 - x - 12)}{(x^2 - 2x - 8)}$, find the hole, the vertical asymptote, the horizontal asymptote, the x -intercept, and the y -intercept.

Problem 20 Designing a rational function (hard)

Write a rational function with a vertical asymptote at $x = 4$, a hole at $x = -3$, a horizontal asymptote $y = 5$, and an x -intercept at 7. Give the hole's coordinates.