

Sequences and Series: Patterns That Add Up

Grade 11 – Algebra 2 math · Lesson · Standards: A-SSE.B.4, F-BF.A.2, F-IF.A.3

Quick answer: An arithmetic sequence adds a constant d to each term, so $a_n = a_1 + (n - 1)d$, while a geometric sequence multiplies by a constant ratio r , so $a_n = a_1 \cdot r^{(n-1)}$. A series is a sum of terms. Finite sums are $n(a_1 + a_n)/2$ for arithmetic and $a_1(1 - r^n)/(1 - r)$ for geometric series.

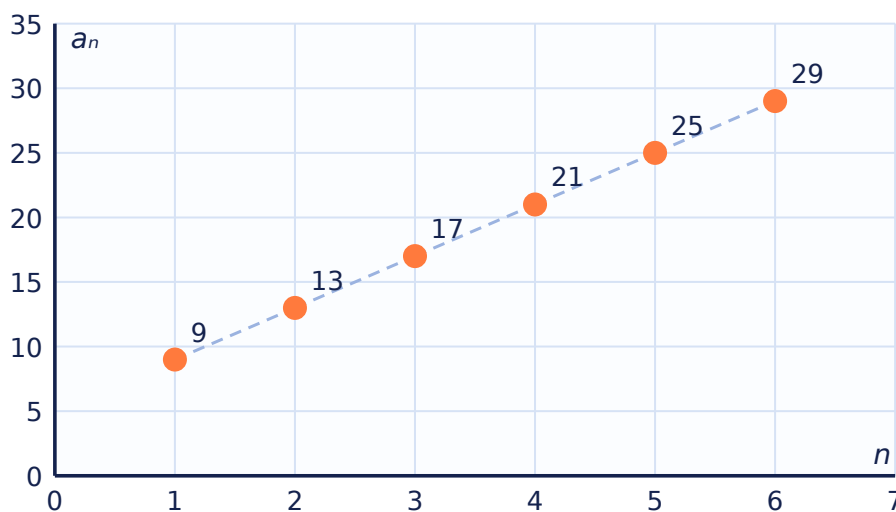
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What are arithmetic and geometric sequences?

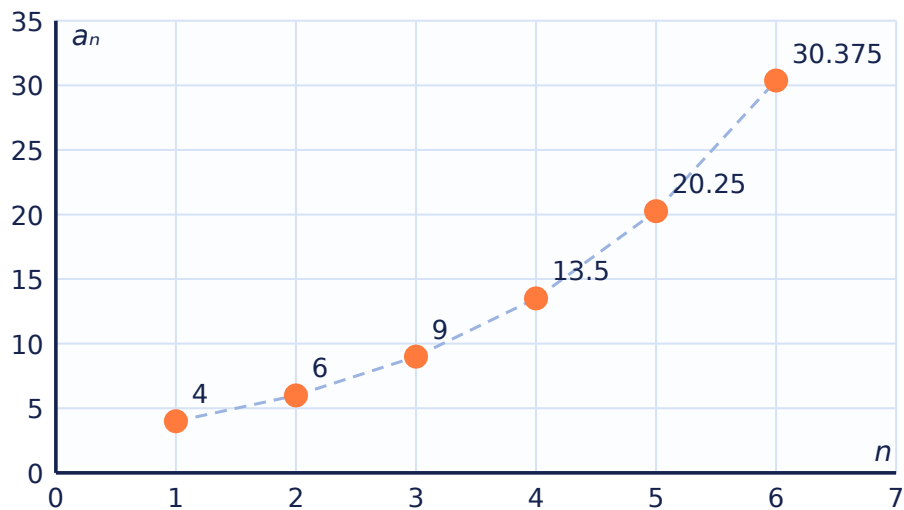
Think of a sequence as a numbered list. Move from one term to the next by adding a fixed d (arithmetic) or by multiplying by a fixed r (geometric).

Rules for the n th term

Type	Explicit	Recursive
Arithmetic	$a_n = a_1 + (n - 1)d$	$a_n = a_{n-1} + d$
Geometric	$a_n = a_1 \cdot r^{n-1}$	$a_n = r \cdot a_{n-1}$



An arithmetic sequence with $d = 4$ plots as points on a line.



A geometric sequence with $r = 1.5$ curves upward.

The recursive rule needs a starting value, such as $a_1 = 9$. Explicit rules jump straight to any term.

What does sigma notation mean?

The symbol $\sum$ means “add up.” The index below it gives the first value, the number above gives the last, and the expression is evaluated for each whole number between them.

$$\sum_{k=1}^5 (3k - 2)$$

last value of k

first value of k

$$= 1 + 4 + 7 + 10 + 13 = 35$$

Sigma notation names the rule, the starting index and the last index.

How do you add the terms of a series?

Sum formulas for the first n terms

- Arithmetic: $S_n = \frac{n(a_1 + a_n)}{2}$
- Geometric: $S_n = a_1 \cdot \frac{(1 - r^n)}{(1 - r)} \quad (r \neq 1)$

Method

1. Identify the type: constant difference or constant ratio.
2. Find n by solving the explicit rule if it is not given.
3. Substitute into the matching sum formula.

Example

Find the sum $4 + 6 + 9 + 13.5 + 20.25 + 30.375$. The ratio is $6 \div 4 = 1.5$, $n = 6$. $S_6 = 4 \cdot \frac{(1 - 1.5^6)}{(1 - 1.5)} = 4 \cdot \frac{(-10.390625)}{(-0.5)} = \mathbf{83.125}$.

Common mistake

Using n instead of $n - 1$ in the exponent of the explicit geometric rule. The first term has exponent 0.

Where do series appear in money problems?

Saving the same amount each month earns interest on every deposit, so the balance is a geometric series. A loan payment is found by setting the sum of the discounted payments equal to the amount borrowed.

Mathbot's tip

Before using any formula, write the first three terms. If you can see what happens each step, you will pick the right formula.

Key points to remember

- Arithmetic: constant difference d ; geometric: constant ratio r .
- Explicit rules give any term directly; recursive rules need the previous term and a starting value.
- Sigma notation $\sum$ adds an expression from a starting index to an ending index.
- Arithmetic sum: $S_n = n(a_1 + a_n)/2$.
- Geometric sum: $S_n = a_1(1 - r^n)/(1 - r)$, with $r \neq 1$.

Sequences and Series: Patterns That Add Up: frequently asked questions

How do I tell whether a sequence is arithmetic or geometric?

Subtract consecutive terms. If the differences are all equal it is arithmetic. If not, divide consecutive terms. If the ratios are all equal it is geometric. Some sequences are neither.

What is the difference between a sequence and a series?

A sequence is a list of terms, such as 3, 7, 11. A series is the sum of those terms, such as $3 + 7 + 11 = 21$. Sigma notation describes a series.

Why is the exponent $n - 1$ in the geometric rule?

The first term has not been multiplied by r yet, so it carries r^0 . The second term has been multiplied

once, giving r^1 , and the n th term has been multiplied $n - 1$ times.

How do I find the number of terms n ?

Set the explicit rule equal to the last term and solve for n . For a geometric sequence use logarithms if the unknown is in an exponent. The result must be a whole number.

How are series used for savings plans?

Each monthly deposit grows for a different number of months. The balances form a geometric series with ratio $1 + i$, where i is the monthly rate, and the geometric sum formula gives the total.