

Experiments, Simulation and Smart Decisions

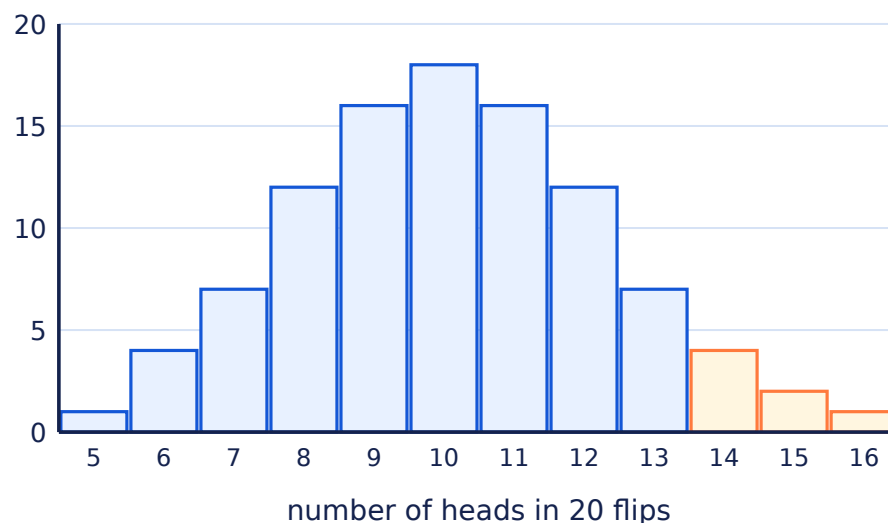
Grade 11 – Algebra 2 math · Lesson · Standards: S-IC.A.2, S-IC.B.5, S-IC.B.6, S-MD.B.6, S-MD.B.7

Quick answer: Simulation tests whether a model fits data: repeat the random process many times and see how often a result as extreme as the observed one occurs. Re-randomizing group labels shows whether a difference between treatments could easily be due to chance. Expected value, the sum of payoff times probability, helps compare plans.

Standards: S-IC.A.2, S-IC.B.5, S-IC.B.6, S-MD.B.6, S-MD.B.7 · **Level:** Grade 11 Algebra 2

How can a simulation test whether a model fits the data?

A model says how likely outcomes are. To test it, simulate many runs of the model with a random device and see how often a result as extreme as the real one appears. If it appears often, the data are consistent with the model. If it almost never appears, the model is doubtful.



Number of heads in 100 simulated runs of 20 fair-coin flips.

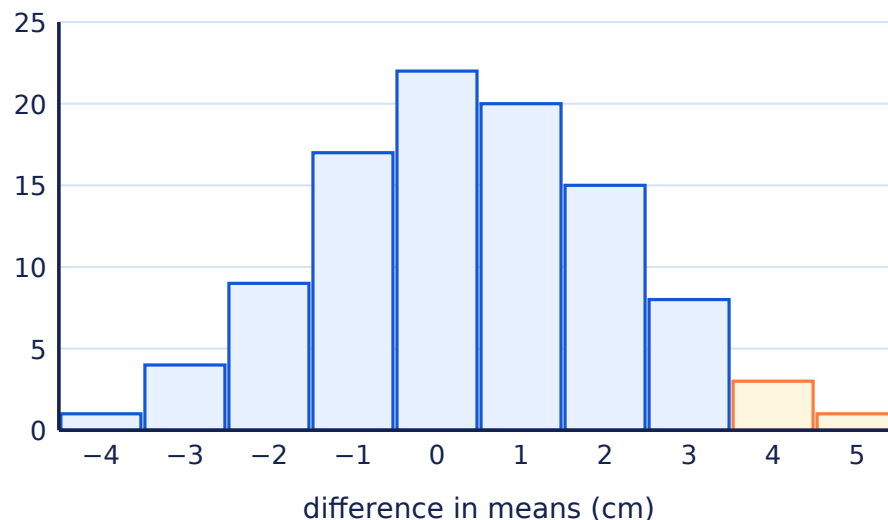
Example

A coin lands heads 14 times in 20 flips. In the 100 simulated runs above, 14 or more heads occurred $4 + 2 + 1 = 7$ times. A 7% chance is not rare enough to rule out a fair coin.

How do you compare two treatments in an experiment?

In a randomized experiment, subjects are assigned to treatments by chance. Record the difference between the group means. Then shuffle the group labels many times, recompute the difference

each time, and see how often chance alone produces a difference as large as the observed one.



Differences in mean growth (cm) from 100 random re-assignments of the 30 plants.

Method

1. State the observed difference in means.
2. Re-randomize the labels many times and record each difference.
3. Count how many simulated differences are at least as large as the observed one.
4. A small fraction (under about 5%) means the difference is statistically significant.

How do you evaluate a data-based report?

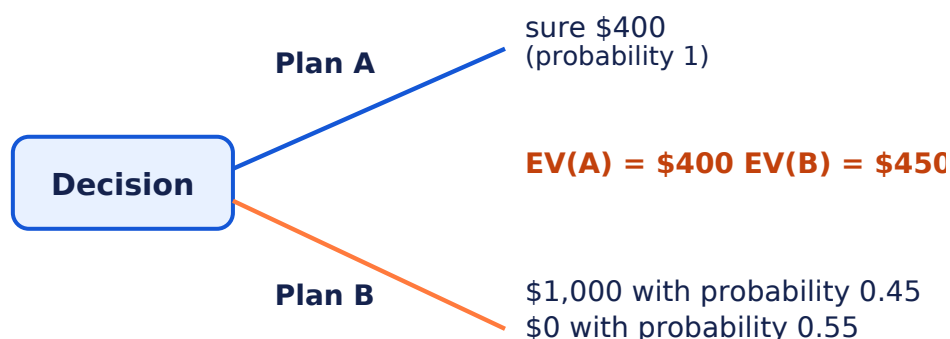
Ask how the data were collected, how large and how random the sample was, whether groups were assigned by chance, and whether the claim goes beyond the data. Only a randomized experiment supports “causes.” Volunteer polls and observational studies can show association only.

Common mistake

Concluding that a result “proves” a claim. A simulation shows how surprising a result is under a model, never certainty.

How does probability support fair decisions?

Random devices such as dice or random digits give each choice an equal chance. The expected value of a plan is the sum of each payoff times its probability.



Expected value compares the long-run average payoff of each plan.

Mathbot's tip

Write the outcomes and probabilities in a small table before computing an expected value. Check that the probabilities sum to 1.

Key points to remember

- A result that appears in under about 5% of simulations is unlikely under the model.
- Random assignment in an experiment lets differences be attributed to the treatment.
- Re-randomizing group labels shows how large differences are by chance alone.
- Reports should be judged by how data were collected, sample size and whether the claim exceeds the data.
- Expected value = sum of (payoff $\times$ probability); a fair game has expected net value 0.

Experiments, Simulation and Smart Decisions: frequently asked questions

What does it mean for a result to be statistically significant?

It means a difference as large as the one observed would be rare if only chance were at work, often fewer than 5 times in 100 simulations. It does not prove the effect is large or important.

Why is random assignment important in an experiment?

Randomly assigning subjects makes the groups similar in every other respect on average. Any big difference in the response can then be attributed to the treatment, not to some hidden factor.

How many simulation runs should I use?

More runs give a steadier estimate. By hand, 100 runs show the pattern, while a computer can do thousands. The estimated probability is the count of extreme results divided by the number of runs.

What is expected value?

It is the long-run average payoff of a random situation: multiply each possible payoff by its probability and add. It does not describe a single trial, only the average over many.

How can I tell a report is unreliable?

Warning signs are a voluntary or convenience sample, a tiny sample, a leading question, an observational study claiming cause and effect, and a margin of error that is missing.