

Experiments, Simulation and Smart Decisions

Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: S-IC.A.2, S-IC.B.5, S-IC.B.6, S-MD.B.6, S-MD.B.7

Each solution counts the simulated runs, converts to a percent, and compares with the 5% cutoff. For expected value problems, check that you multiplied each payoff by its probability.

Standards: S-IC.A.2, S-IC.B.5, S-IC.B.6, S-MD.B.6, S-MD.B.7 · **Level:** Grade 11 Algebra 2

Problem 1 Assign random digits

Let the digits 0, 1, 2, 3, 4, 5 (six of ten) stand for an in-bounds serve and 6, 7, 8, 9 for a fault. Then an in-bounds serve has probability 6/10. **Digits 0-5 = in bounds, 6-9 = fault.**

Problem 2 Run a simulation by hand

Digits 0-6 are makes: 3, 1, 0, 6, 2, 4, 5 are makes (7 of them); 7, 8, 9 are misses. **7 makes in 10 shots.**

Problem 3 Estimate a probability from trials

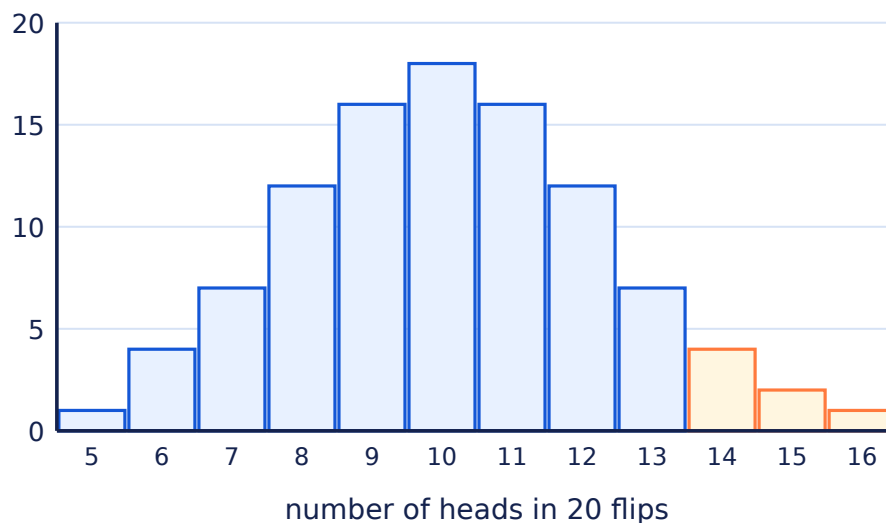
$46 \div 200 = 0.23. \approx \mathbf{0.23}.$

Problem 4 Compare a simulation with theory

Experimental: $7/50 = 0.14$. Theoretical: $(1/2)(1/2)(1/2) = 1/8 = 0.125$. **0.14 compared with 0.125: close, and the gap is the normal variation of a small simulation.**

Problem 5 Is the coin fair? (14 heads)

The bars for 14, 15 and 16 heads show 4, 2 and 1 runs: 7 of 100 runs, or 7%. A result this extreme happens about 7% of the time with a fair coin, which is not rare. **7 runs; a fair coin is plausible.**



The shaded bars are the runs with 14 or more heads.

Problem 6 Is the coin fair? (18 heads)

No bar extends past 16 heads, so 0 of the 100 runs had 18 or more. **0 runs; 18 heads is very unlikely with a fair coin, so the coin is probably biased.**

Problem 7 Parts of an experiment

Treatments: energy drink and plain water (the water group is the control). Response variable: reaction time. Random assignment makes the two groups alike in other ways, so a difference can be credited to the drink. **Treatments: drink or water; response: reaction time; random assignment balances other factors.**

Problem 8 Correlation and causation

No. Students chose whether to eat breakfast, so this is an observational study. Other factors, such as sleep or family routine, may affect both breakfast and GPA. **No: only a randomized experiment can support cause and effect.**

Problem 9 Evaluate a poll

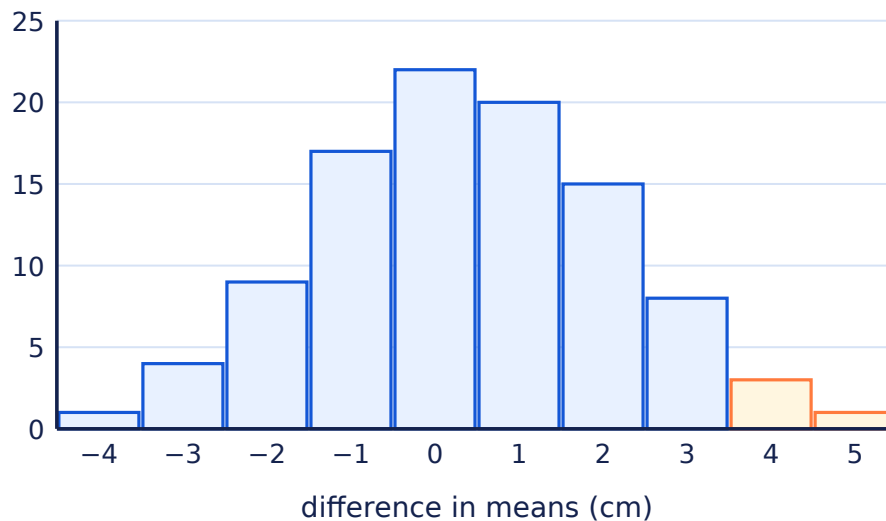
The respondents chose themselves, so fans of the restaurant are over-represented (voluntary response bias). The question is also leading. **Self-selected sample and a leading question.**

Problem 10 Sample size and margin of error

For 100: $1/10 = 0.10$. For 1000: $1/\sqrt{1000} = 0.0316$. $\approx$ **0.10 (10 points) and $\approx$ 0.032 (3.2 points)**: ten times the sample size cuts the margin by about a factor of 3.

Problem 11 Re-randomization for two fertilizers

The bars for 4 and 5 cm show $3 + 1 = 4$ runs out of 100. A difference this large occurred by chance only 4% of the time. **4 of 100 runs (4%).**



The shaded bars are differences of 4 cm or more.

Problem 12 Draw a conclusion from the simulation

Since 4% is below 5%, a difference of 4 cm would be unusual by chance alone. **Yes: there is evidence that fertilizer A produces more growth than fertilizer B**, and because the plants were randomly assigned, the fertilizer can be credited.

Problem 13 Choose fairly among three options

Assign 1 and 2 to Ana, 3 and 4 to Ben, and 5 and 6 to Cara. Roll once. Each friend has $\frac{2}{6} = \frac{1}{3}$ chance. **Each has probability $\frac{1}{3}$.**

Problem 14 Is the game fair?

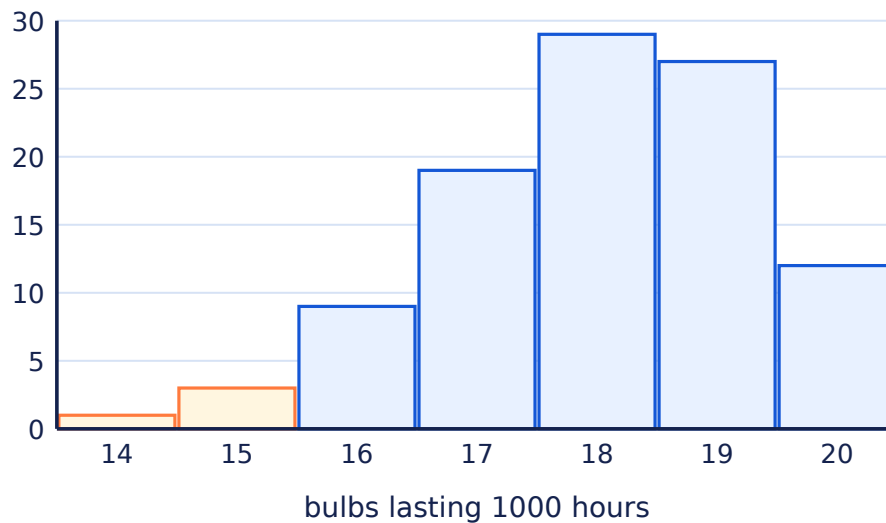
Expected winnings = $9(\frac{1}{6}) + 0(\frac{5}{6}) = \$1.50$. Net = $1.50 - 2 = -\$0.50$. **Expected net $-\$0.50$ per play, so the game is not fair.**

Problem 15 Raffle expected value

Total prizes = $200 + 2(50) + 5(20) = 400$, so the expected prize per ticket is $\frac{400}{500} = \$0.80$. Net = $0.80 - 5 = -\$4.20$. **Expected prize $\$0.80$; expected net $-\$4.20$.**

Problem 16 Test a manufacturer's claim

In the simulation, 15 or fewer long-lasting bulbs occurred $3 + 1 = 4$ times in 100, or 4%. That is unusual if the claim were true. **The claim is doubtful (a result this low happens about 4% of the time).**



The shaded bars show samples with 15 or fewer long-lasting bulbs.

Problem 17 Warranty decision

Expected repair cost = $0.08(450) = \$36$. The warranty costs \$60, which is \$24 more than the expected cost. **On average the warranty loses \$24, so skipping it is better in expectation** (though it removes the risk of a large bill).

Problem 18 Evaluate a tutoring report

Possible questions: (1) Were students assigned at random, or did they choose to enroll? (2) Was there a comparison group that did not receive tutoring? (3) Is a sample of 12 large enough to separate the 5-point rise from chance? **Self-selection, no control group and a small sample all limit the conclusion.**

Problem 19 Sure thing versus gamble

Plan A: \$550. Plan B: $1000(0.6) + 0(0.4) = \$600$. **Plan B has the higher expected value (\$600 versus \$550)**, but it carries the risk of \$0.

Problem 20 Compare two machines

Machine X: $0.92(3) - 0.08(5) = 2.76 - 0.40 = \$2.36$. Machine Y: $0.97(2.50) - 0.03(4) = 2.425 - 0.12 = \$2.305$. **Machine X, with \$2.36 versus about \$2.31 per item.**