

# Structure, Identities and Modeling with Equations

**Grade 11 – Algebra 2 math · Worksheet Answer Key · Standards: A-SSE.A.1, A-SSE.A.2, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.D.11, A-APR.C.5, N-CN.C.8, N-CN.C.9**

Every solution shows the pattern or equation used and a check. When a model has constraints, verify that your answer satisfies them as well as the equation.

**Standards:** A-SSE.A.1, A-SSE.A.2, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.D.11, A-APR.C.5, N-CN.C.8, N-CN.C.9 · **Level:** Grade 11 Algebra 2

## Problem 1 Interpret a model

500 is the starting amount in dollars (the value when  $t = 0$ ). The factor 1.04 means the balance is multiplied by 1.04 each year, a 4% yearly increase. **500 = initial deposit; 1.04 = yearly growth factor (4%).**

## Problem 2 Difference of squares twice

$$x^4 - 625 = (x^2 + 25)(x^2 - 25) = (x^2 + 25)(x + 5)(x - 5). \quad \mathbf{(x^2 + 25)(x + 5)(x - 5)}.$$

## Problem 3 Factor with a coefficient

$$16x^4 = (4x^2)^2 \text{ and } 81 = 9^2, \text{ so the expression is } (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x + 3)(2x - 3). \quad \mathbf{(4x^2 + 9)(2x + 3)(2x - 3)}.$$

## Problem 4 Rearrange a volume formula

$$\text{Multiply by 3: } 3V = \pi r^2 h. \text{ Divide by } \pi r^2. \quad \mathbf{h = 3V/(\pi r^2)}.$$

## Problem 5 Rearrange a trapezoid area formula

$$\text{Multiply by 2: } 2A = h(b_1 + b_2). \text{ Divide by } h: 2A/h = b_1 + b_2. \text{ Subtract } b_1. \quad \mathbf{b_2 = 2A/h - b_1}.$$

## Problem 6 Expand with Pascal's triangle

$$\text{Row 4 is 1, 4, 6, 4, 1 (each entry is the sum of the two above). Then } (x + 2)^4 = x^4 + 4(2)x^3 + 6(4)x^2 + 4(8)x + 16. \quad \mathbf{x^4 + 8x^3 + 24x^2 + 32x + 16}.$$

Row 4 is 1, 4, 6, 4, 1.

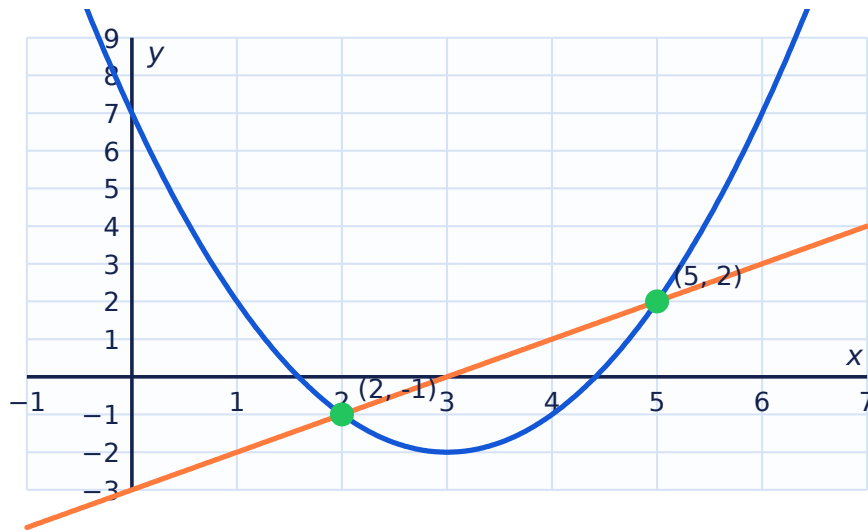
Row 3 is 1, 3, 3, 1.  $(2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3 = 8x^3 - 12x^2 + 6x - 1$ .  **$8x^3 - 12x^2 + 6x - 1$ .**

The term is  $C(6, 2) \cdot x^2 \cdot 2^4 = 15 \cdot 16 \cdot x^2$ . **240.**

$$x^2 + 25 = x^2 - (5i)^2 = (x + 5i)(x - 5i). \text{ Check: } x^2 - 25i^2 = x^2 + 25. \text{ (x + 5i)(x - 5i).}$$
$$x = (4 \pm \sqrt{(16 - 52)})/2 = (4 \pm 6i)/2 = 2 \pm 3i. \text{ **(}x - 2 - 3i)(x - 2 + 3i)\text{.}** Check: } (x - 2)^2 + 9 = x^2 - 4x + 13.$$

Right side:  $x^4 - 2x^2y^2 + y^4 + 4x^2y^2 = x^4 + 2x^2y^2 + y^4$ , the left side. For (4, 1):  $17^2 = 15^2 + 8^2$ , since  $289 = 225 + 64$ . **Triple 8, 15, 17.**

The curves cross at  $x = 2$  and  $x = 5$ . Algebraically,  $x^2 - 6x + 7 = x - 3$  gives  $x^2 - 7x + 10 = 0$ , so  $(x - 2)(x - 5) = 0$ .  **$x = 2$  or  $x = 5$ .**



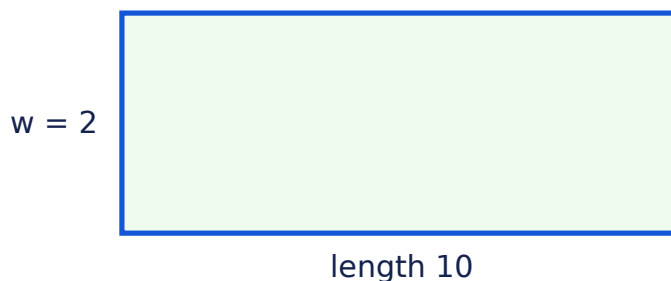
The intersections are  $(2, -1)$  and  $(5, 2)$ .

### Problem 13 Solve with a table

Compare columns: at  $x = 2$  both are 10, and at  $x = 3$  both are 15. In the other columns they differ.  **$x = 2$  and  $x = 3$ .**

### Problem 14 Garden constraint

The length is  $12 - w$ , so the area is  $w(12 - w) \geq 20$ . Then  $w^2 - 12w + 20 \leq 0$ , so  $(w - 2)(w - 10) \leq 0$ .  **$2 \leq w \leq 10$  feet** (every width in this interval gives area at least 20).



At the boundary, 2 ft by 10 ft has area 20 ft<sup>2</sup>.

### Problem 15 Depreciation model

$V(t) = 24000(0.88)^t$ . Set  $24000(0.88)^t = 15000$ :  $0.88^t = 0.625$ , so  $t = \ln 0.625 \div \ln 0.88 = 3.677$ .  
 **$V(t) = 24000(0.88)^t$ ;  $\approx 3.7$  years.**

### Problem 16 Mixture as a rational equation

Solve  $(4 + x)/(20 + x) = 0.5$ . Cross-multiply:  $4 + x = 10 + 0.5x$ , so  $0.5x = 6$ .  **$x = 12$  liters.**  
Check:  $16/32 = 0.5$ .

### Problem 17 Ticket sales system

Let  $x$  be adult tickets and  $y$  child tickets:  $x + y = 150$  and  $12x + 7y = 1330$ . Substitute  $y = 150 - x$ :  $12x + 1050 - 7x = 1330$ , so  $5x = 280$ . **56 adult and 94 child tickets**. Check:  $672 + 658 = 1330$ .

### Problem 18 Budget constraints

Let  $x$  = notebooks and  $y$  = pens:  $4x + 1.5y \leq 60$ ,  $x \geq 6$ ,  $y \geq 0$ . Test (6, 20):  $4(6) + 1.5(20) = 24 + 30 = 54 \leq 60$  and  $6 \geq 6$ . **Yes, (6, 20) is allowed.**

### Problem 19 Binomial Theorem with complex numbers

Row 4 is 1, 4, 6, 4, 1:  $1 + 4i + 6i^2 + 4i^3 + i^4 = 1 + 4i - 6 - 4i + 1$ . **-4.**

### Problem 20 Identities for mental arithmetic

$(144 + 25)(12 + 5)(12 - 5) = 169 \cdot 17 \cdot 7 = 169 \cdot 119 = 20,111$ . Check:  $20,736 - 625 = 20,111$ . **20,111.**