

Using Derivatives to Analyze Graphs and Optimize

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 5

Worked solutions and scoring guides follow. Credit a correct justification even if arithmetic slips, but require a sign chart or theorem statement where the question asks for justification, since an unsupported answer earns only partial credit.

Standards: AP Calculus AB Unit 5 · **Level:** Grade 12 AP Calculus AB

Question 1 Absolute maximum (2 points)

$f(x) = 3x^2 - 12 = 0$ gives $x = \pm 2$. Candidates: $f(-3) = 9$, $f(-2) = 16$, $f(2) = -16$, $f(5) = 65$. The maximum is 65. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for listing the candidates.

Question 2 Local maximum from a graph (2 points)

$f(x)$ is negative for $x < -2$, positive on $(-2, 1)$ and negative for $x > 1$. The sign changes from + to – at $x = 1$. Answer **(C)**.

Scoring: 2 points for the correct choice; 1 point for identifying where $f(x)$ changes sign.

Question 3 Mean Value Theorem application (3 points)

(a) f is a polynomial, so it is continuous on $[0, 3]$ and differentiable on $(0, 3)$.

(b) The average rate is $(f(3) - f(0))/3 = (21 - 0)/3 = 7$. Solve $3c^2 - 2 = 7$, so $c^2 = 3$ and $c = \sqrt{3} \approx 1.732$ (the negative root is outside $(0, 3)$).

Scoring: Part (a): 1 point. Part (b): 1 point for the average rate, 1 point for c .

Question 4 Concavity analysis (3 points)

(a) $g' = 4x^3 - 24x^2 + 36x$, so $g'' = 12x^2 - 48x + 36 = 12(x - 1)(x - 3)$.

(b) $g'' > 0$ for $x < 1$ and $x > 3$, and < 0 on $(1, 3)$. So **concave up on $(-\infty, 1)$ and $(3, \infty)$, concave down on $(1, 3)$.**

(c) The sign of g'' changes at $x = 1$ and $x = 3$, so those are the **inflection points**, with $g(1) = 11$ and $g(3) = 27$.

Scoring: 1 point per part.

Question 5 Minimizing the surface of a box (3 points)

(a) The height is $h = 1,728/s^2$, so $S = 2s^2 + 4sh = 2s^2 + 6,912/s$.

(b) $S' = 4s - 6,912/s^2 = 0$ gives $s^3 = 1,728$, so $s = 12$ cm (and $h = 12$).

(c) $S'' = 4 + 13,824/s^3 > 0$ for $s > 0$, so the graph is concave up and $s = 12$ gives a minimum.

Scoring: 1 point per part.

Question 6 First derivative test (3 points)

(a) $f(x) = 6x^2 - 18x + 12 = 6(x - 1)(x - 2)$.

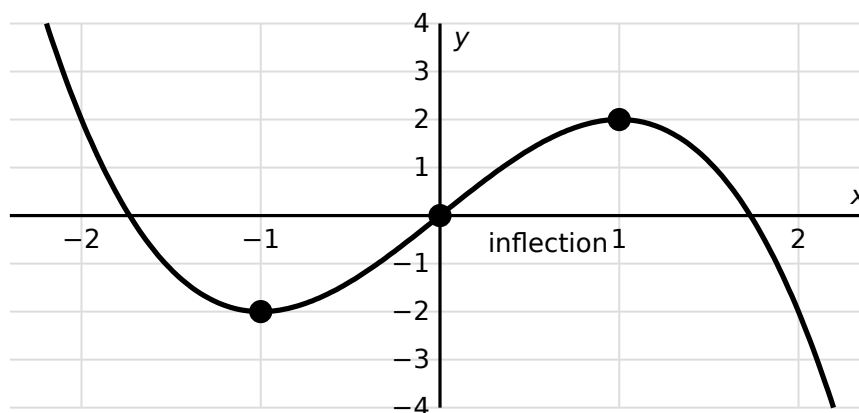
(b) Signs: positive for $x < 1$, negative on $(1, 2)$, positive for $x > 2$. So $x = 1$ is a **local maximum** and $x = 2$ a **local minimum**.

(c) $f(1) = 4$ and $f(2) = 3$. The **local maximum value is 4 and the local minimum value is 3**.

Scoring: 1 point per part.

Question 7 Analyzing a graph (2 points)

The graph rises between the minimum at $x = -1$ and the maximum at $x = 1$, so f is **increasing on $(-1, 1)$** . It bends upward to the left of the origin and downward to the right, so f is **concave up on $(-\infty, 0)$** , which on the graph shown is the stretch from the left edge to the origin.



The concavity changes at the origin.

Scoring: 1 point for each interval.

Question 8 Extreme Value Theorem hypotheses (2 points)

The theorem needs continuity on the closed interval. g is not continuous at $x = 1$, where it has a vertical asymptote, so **the theorem does not apply**. Indeed, $g(x)$ becomes arbitrarily large as $x \rightarrow 1^+$, so **g has no absolute maximum** on $[0, 2]$.

Scoring: 1 point for the discontinuity, 1 point for the conclusion about the maximum.