

Using Derivatives to Analyze Graphs and Optimize

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 5

Name: _____ Date: _____ Class: _____

This 45-minute test follows the AP Calculus AB style: two multiple-choice items with options and free-response questions that require justification. Show all work, including sign analysis. The total is 20 points, and no calculator is needed.

Standards: AP Calculus AB Unit 5 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

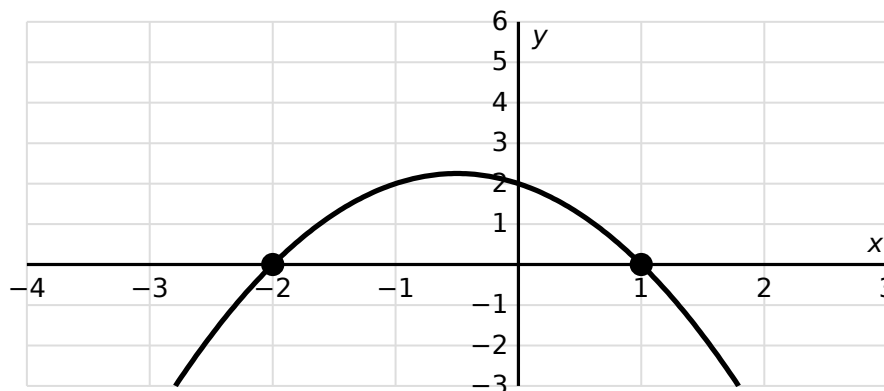
Question 1 Absolute maximum (2 points)

What is the absolute maximum value of $f(x) = x^3 - 12x$ on $[-3, 5]$?

- A. 16
- B. 65
- C. 9
- D. 5

Question 2 Local maximum from a graph (2 points)

The graph of $f(x)$ is shown. At which x -value does f have a local maximum?



Graph of $f(x)$.

- A. -2

- B. 0
- C. 1
- D. There is none

Question 3 Mean Value Theorem application (3 points)

Let $f(x) = x^3 - 2x$ on $[0, 3]$.

- a. Verify that the hypotheses of the Mean Value Theorem are satisfied. (1 point)
- b. Find all values c that satisfy the conclusion. (2 points)

Question 4 Concavity analysis (3 points)

Let $g(x) = x^4 - 8x^3 + 18x^2$.

- a. Find $g''(x)$. (1 point)
- b. Find the intervals where g is concave up and concave down. (1 point)
- c. Find the inflection points and justify. (1 point)

Question 5 Minimizing the surface of a box (3 points)

A closed box with a square base has volume $1,728 \text{ cm}^3$. Let s be the base side.

- a. Write the surface area S as a function of s alone. (1 point)
- b. Determine the base side that makes S smallest. (1 point)
- c. Justify that it is a minimum. (1 point)

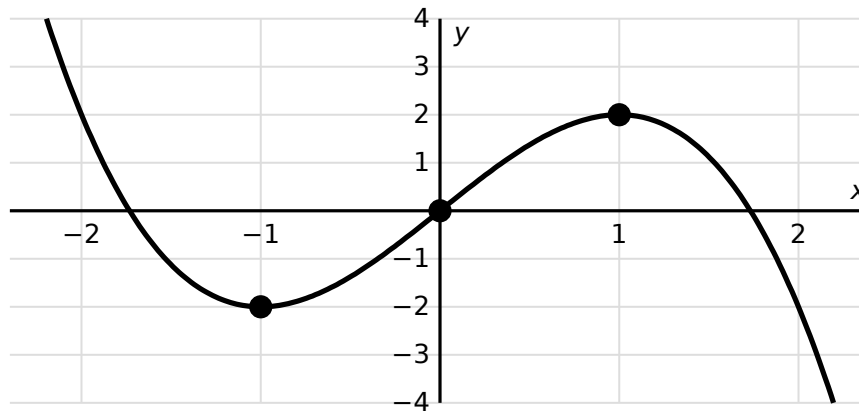
Question 6 First derivative test (3 points)

Let $f(x) = 2x^3 - 9x^2 + 12x - 1$.

- a. Find $f'(x)$. (1 point)
- b. Use a sign chart for $f'(x)$ to classify each critical number. (1 point)
- c. Find the local maximum and minimum values. (1 point)

Question 7 Analyzing a graph (2 points)

The graph of f is shown. Give the interval on which f is increasing and the interval on which f is concave up.



Graph of f .

Question 8 Extreme Value Theorem hypotheses (2 points)

Does the Extreme Value Theorem guarantee an absolute maximum for $g(x) = 1/(x - 1)$ on $[0, 2]$? Explain, and state whether g actually has an absolute maximum.