

Using Derivatives to Analyze Graphs and Optimize

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 5

Name: _____ Date: _____ Class: _____

These 20 problems cover AP Calculus AB Unit 5: critical numbers, increasing and decreasing intervals, concavity, extrema tests, the Mean Value and Extreme Value Theorems, curve sketching and optimization. Early problems apply a single test; later ones need several.

The last six problems are applied or multi-step, including a fence, a cardboard box and a drone, where a model must be built before it is differentiated.

Standards: AP Calculus AB Unit 5 · **Level:** Grade 12 AP Calculus AB

Problem 1 Absolute extrema on a closed interval (easy)

Find the absolute maximum and minimum of $f(x) = x^3 - 3x^2 - 9x + 5$ on $[-2, 4]$.

Problem 2 Increasing and decreasing intervals (easy)

Find where $g(x) = x^4 - 8x^2$ is increasing and where it is decreasing.

Problem 3 First Derivative Test with an exponential (medium)

Let $h(x) = xe^{-2x}$. Find its local extremum and classify it.

Problem 4 Concavity and inflection points (medium)

Let $f(x) = x^4 - 6x^2 + 4x$. Find the intervals of concavity and the inflection points (give x-values).

Problem 5 Second Derivative Test (medium)

Classify the critical points of $f(x) = x^3 - 12x + 7$ using the sign of the second derivative.

Problem 6 Mean Value Theorem (medium)

Verify the hypotheses of the Mean Value Theorem for $f(x) = x^2 - 6x + 1$ on $[1, 7]$ and find every guaranteed value c .

Problem 7 When the Mean Value Theorem does not apply (medium)

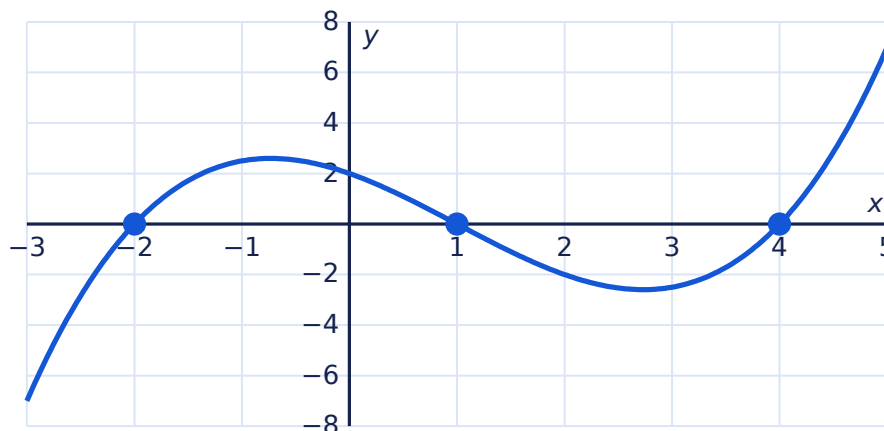
Let $f(x) = |x - 2|$ on $[0, 5]$. Explain which hypothesis of the Mean Value Theorem fails, and show that no number c has the matching slope.

Problem 8 Locating an inflection point (medium)

Find the inflection point of $f(x) = 2x^3 - 15x^2 + 36x$.

Problem 9 Reading the graph of a derivative (medium)

The graph of $f'(x)$ is shown. On which intervals is f increasing? Where does f have local maxima and minima?



Graph of $f'(x)$.

Problem 10 Inflection points from a derivative formula (hard)

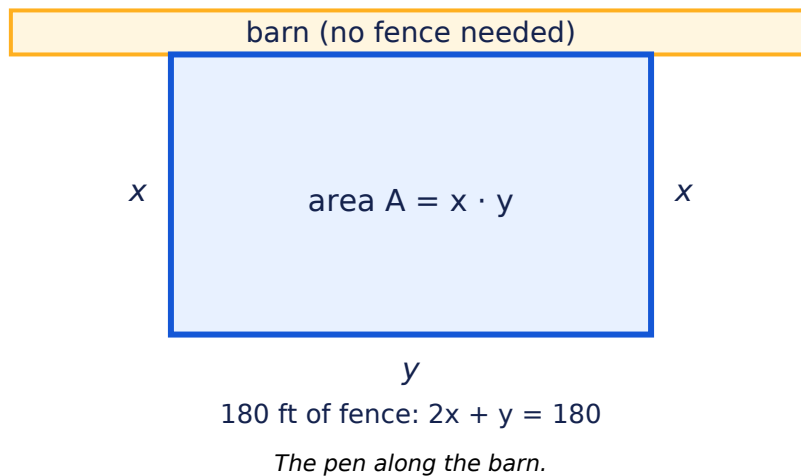
If $f'(x) = \frac{(x+2)(x-1)(x-4)}{4}$, find the x -coordinates of the inflection points of f .

Problem 11 Absolute extrema with a reciprocal (hard)

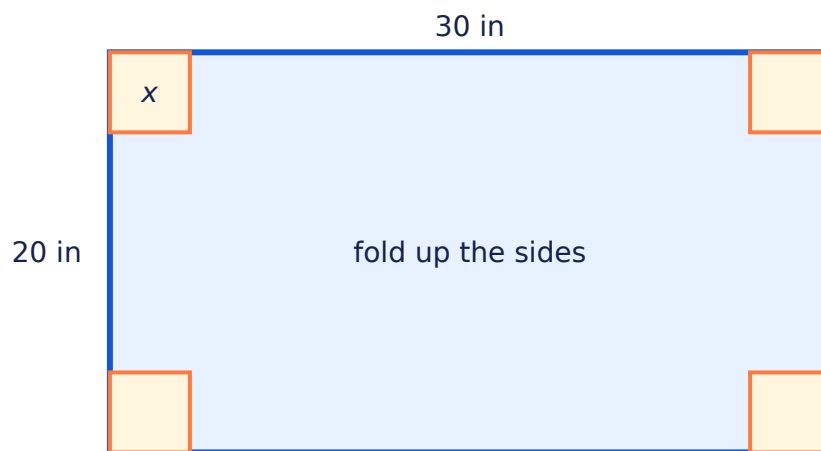
Find the absolute extrema of $f(x) = x + \frac{4}{x}$ on $[1, 5]$.

Problem 12 Fencing a pen against a barn (hard)

A farmer has 180 feet of fence to enclose a rectangular pen. One side is a barn and needs no fence. Find the dimensions that maximize the area and state the maximum area.

**Problem 13 Open box from cardboard** (hard)

Squares of side x are cut from the corners of a 30-inch by 20-inch sheet and the sides are folded up to form an open box. Find x that maximizes the volume, and the maximum volume to the nearest cubic inch.



Cardboard sheet with corner squares.

Problem 14 Max and min on an ellipse (hard)

On the ellipse $x^2 + 4y^2 = 16$, find the points where the tangent line is horizontal and use the second derivative to classify them.

Problem 15 Connecting f , f' and f'' (hard)

A continuous function f has $f'(x) > 0$ on $(-\infty, -1)$, $f'(x) < 0$ on $(-1, 3)$, $f'(x) > 0$ on $(3, \infty)$, and $f'' < 0$ on $(-\infty, 1)$, $f'' > 0$ on $(1, \infty)$. Locate all local extrema and inflection points and describe the sketch.

Problem 16 Ticket price and revenue (hard)

A theater finds that at a ticket price of p dollars it sells $500 - 20p$ tickets. The price must be between \$5 and \$20. Use the Candidates Test to find the price that maximizes revenue.

Problem 17 Minimizing the surface of a can (hard)

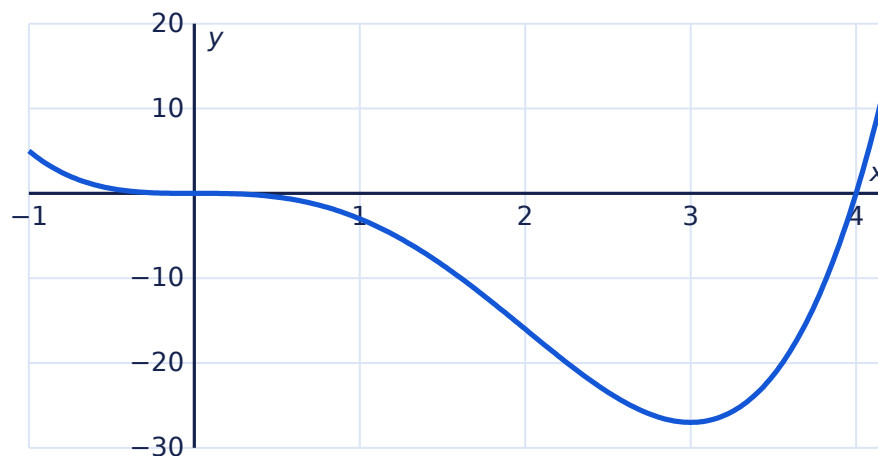
A cylindrical can must hold 500 cm^3 . Determine r and h giving the least surface area, to the nearest 0.1 cm.

Problem 18 Mean Value Theorem for a drone (hard)

A drone's altitude $h(t)$, in meters, is differentiable on $[0, 8]$ with $h(0) = 15$ and $h(8) = 95$. Must the drone have a vertical speed of exactly 10 m/s at some moment? Must it have exactly 12 m/s?

Problem 19 Complete analysis of a quartic (hard)

Analyze $f(x) = x^4 - 4x^3$: find critical numbers, intervals of increase, local extrema, concavity and inflection points.



Graph of f .

Problem 20 Finding coefficients from conditions (hard)

Let $f(x) = x^3 + ax^2 + bx + c$. The graph has an inflection point at $x = 2$ and a critical number at $x = -1$, and $f(0) = -7$. Find a , b , c and classify both critical points.