

Applications of Integration: Area and Volume

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 8

Worked solutions and scoring guides follow. Award setup points for a correct integrand and limits even if the final evaluation contains an arithmetic slip, but require units where the question is set in context.

Standards: AP Calculus AB Unit 8 · **Level:** Grade 12 AP Calculus AB

Question 1 Average value (2 points)

$\int_1^3 3x^2 dx = 27 - 1 = 26$. Divide by the width 2: 13. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for the integral value 26.

Question 2 Area between curves (2 points)

The curves meet at $x = 0$ and $x = 2$, with the line on top. Area = $\int_0^2 (2x - x^2) dx = 4 - 8/3 = 4/3$. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for finding the limits.

Question 3 Disc method (3 points)

(a) The disc radius is x^3 , so $V = \pi \int_1^2 (x^3)^2 dx = \pi \int_1^2 x^6 dx$.

(b) $V = \pi$ from 1 to 2 = $\pi(128 - 1)/7 = 127\pi/7$.

Scoring: Part (a): 1 point for π and the squared radius, 1 point for the limits. Part (b): 1 point.

Question 4 Washer method (3 points)

(a) $5x = x^2$ gives $x = 0$ and 5 .

(b) The line is the outer curve: $V = \pi \int_0^5 (25x^2 - x^4) dx$.

(c) $\pi[25x^3/3 - x^5/5]$ from 0 to 5 = $\pi(3125/3 - 625) = 1250\pi/3$.

Scoring: 1 point per part.

Question 5 Cross sections (3 points)

(a) The side is $\sqrt{x}$, so $A = (\sqrt{3/4})(x) = (\sqrt{3/4})x$.

(b) $V = \int_0^4 (\sqrt{3/4})x \, dx$.

(c) $(\sqrt{3/4})(16/2) = 2\sqrt{3} \approx 3.46$.

Scoring: 1 point per part.

Question 6 Displacement and distance (3 points)

(a) $\int_0^6 v \, dt = [t^3/3 - 3t^2 + 5t]$ from 0 to 6 = $72 - 108 + 30 = -6$.

(b) $v = (t - 1)(t - 5)$ changes sign at 1 and 5. Positions: $s(1) = 7/3$, $s(5) = -25/3$, $s(6) = -6$.
Distance = $7/3 + 32/3 + 7/3 = 46/3 \approx 15.3$.

Scoring: Part (a): 1 point. Part (b): 1 point for splitting at the sign changes, 1 point for the total.

Question 7 Rates in and out (2 points)

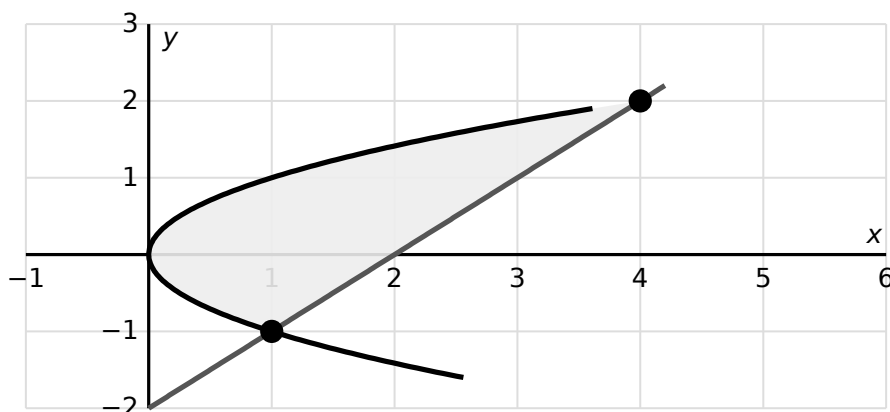
(a) $50 + \int_0^8 (3t - 9) \, dt = 50 + (96 - 72) = 74$ gallons.

(b) At $t = 2$, the inflow is 6 and the outflow is 9, so the net rate is $-3 < 0$ and the amount is **decreasing**.

Scoring: 1 point per part.

Question 8 Area with respect to y (2 points)

The curves meet where $y^2 = y + 2$, so $y = -1$ and $y = 2$. The line is on the right. **Area** = $\int_{-1}^2 (y + 2 - y^2) \, dy = [y^2/2 + 2y - y^3/3]$ from -1 to $2 = 10/3 - (-7/6) = 9/2$.



The shaded region.

Scoring: 1 point for the integral, 1 point for the value.