

# Applications of Integration: Area and Volume

## Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 8

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

This 45-minute test follows the AP Calculus AB style: two multiple-choice items and free-response questions requiring an integral and a value. Show every setup. The total is 20 points, and no calculator is needed.

**Standards:** AP Calculus AB Unit 8 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

### Question 1 Average value (2 points)

What is the average value of  $f(x) = 3x^2$  on  $[1, 3]$ ?

- A. 13
- B. 26
- C. 8
- D. 9

### Question 2 Area between curves (2 points)

Which value equals the area trapped between  $y = 2x$  and  $y = x^2$ ?

- A.  $4/3$
- B.  $8/3$
- C. 2
- D. 4

### Question 3 Disc method (3 points)

The region bounded by  $y = x^3$ , the  $x$ -axis,  $x = 1$  and  $x = 2$  is rotated around the  $x$ -axis.

- a. Write an integral for the volume. (2 points)
- b. Evaluate it. (1 point)

#### Question 4 Washer method (3 points)

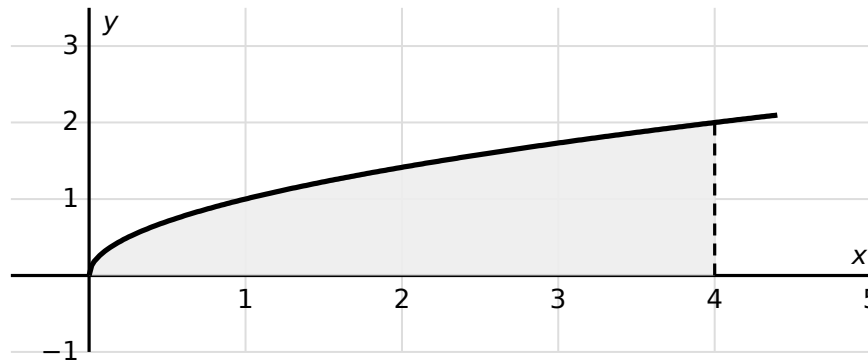
Let  $R$  be the region enclosed by  $y = 5x$  and  $y = x^2$ .

- Find the intersection points. (1 point)
- Write an integral for the volume when  $R$  is rotated around the  $x$ -axis. (1 point)
- Evaluate it. (1 point)

#### Question 5 Cross sections (3 points)

A museum model rests on the base under  $y = \sqrt{x}$  for  $0 \leq x \leq 4$ . Vertical slices taken at right angles to the  $x$ -axis are equilateral triangles standing on the base.

- Find the area of a cross section at  $x$ . (1 point)
- Write an integral for the volume. (1 point)
- Find the volume. (1 point)



The base region.

#### Question 6 Displacement and distance (3 points)

A cart rolls along a track with velocity  $v(t) = t^2 - 6t + 5$  over  $0 \leq t \leq 6$ .

- Find the net change in position. (1 point)
- Find the total path length. (2 points)

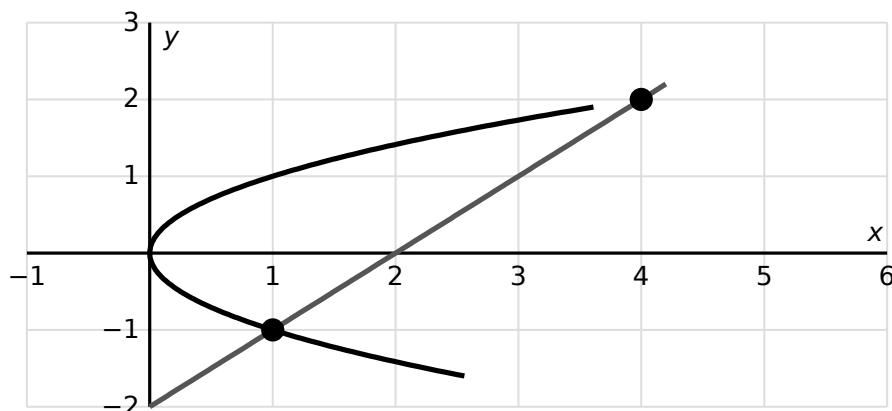
#### Question 7 Rates in and out (2 points)

A reservoir begins with 50 gallons. During the next 8 minutes, a hose adds  $I(t) = 3t$  gallons each minute while a pump removes a steady 9 gallons each minute.

- How many gallons are in the tank at  $t = 8$ ? (1 point)
- Is the amount increasing or decreasing at  $t = 2$ ? Justify. (1 point)

### Question 8 Area with respect to y (2 points)

Two boundaries,  $x = y^2$  and  $x = y + 2$ , enclose the shaded region shown. Set up and evaluate a y-integral for its area.



*The region.*