

Applications of Integration: Area and Volume

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 8

Name: _____ Date: _____ Class: _____

These 20 problems cover AP Calculus AB Unit 8: average value, motion with integrals, area between curves, volumes by cross sections, discs and washers, and accumulation in applied contexts. Early problems use one formula; later ones need a sketch and several steps.

The last six problems are multi-step: curves that cross, a vertical axis of revolution, an average temperature, a tent volume and a combined area and volume problem.

Standards: AP Calculus AB Unit 8 · **Level:** Grade 12 AP Calculus AB

Problem 1 Average value of a polynomial (easy)

Compute the mean height of the graph of $f(x) = 4x^2 - 3x$ between $x = 2$ and $x = 5$.

Problem 2 Average value of sine (easy)

Find the average value of $f(x) = \sin x$ on $[0, \pi]$.

Problem 3 Position from velocity (easy)

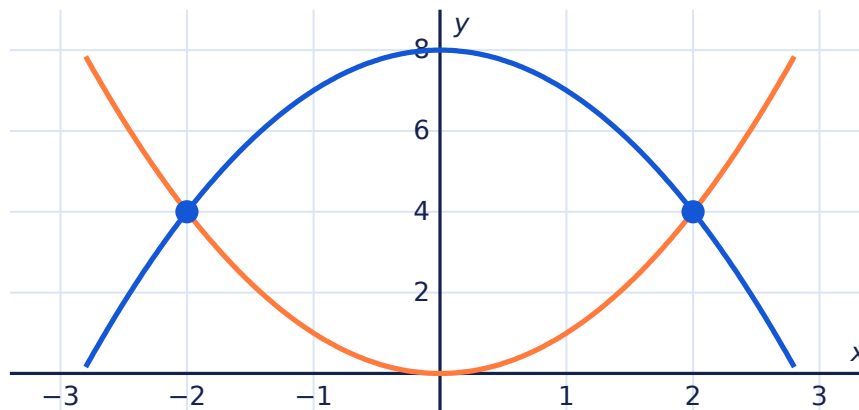
A cart starts at position $s(0) = 5$ meters with velocity $v(t) = 4t - t^2$ m/s. Find its position at $t = 3$.

Problem 4 Net change and total distance (medium)

A delivery drone flies along a straight line with velocity $v(t) = t^2 - 7t + 10$ meters per second for $0 \leq t \leq 6$. Find its net change in position and the full path length.

Problem 5 Area between two parabolas (medium)

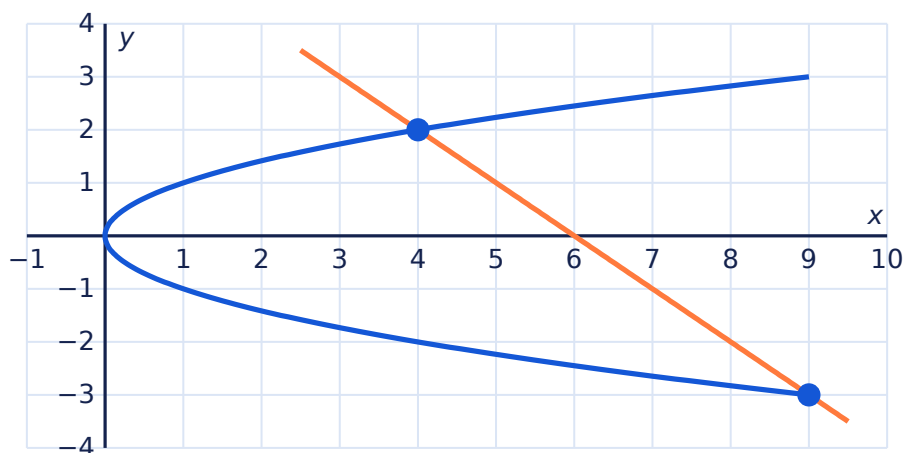
How much area lies between the downward parabola $y = 8 - x^2$ and the upward parabola $y = x^2$?



The two parabolas.

Problem 6 Area integrated with respect to y (medium)

Compute the area of the pocket between the sideways parabola $x = y^2$ and the line $x = 6 - y$.



The two curves.

Problem 7 Square cross sections (medium)

A sculpture has a footprint between the curves $y = 2x$ and $y = x^2$. Slicing it by planes perpendicular to the x -axis gives square slices whose edge spans the footprint. Find the volume of the sculpture.

Problem 8 Semicircular cross sections (medium)

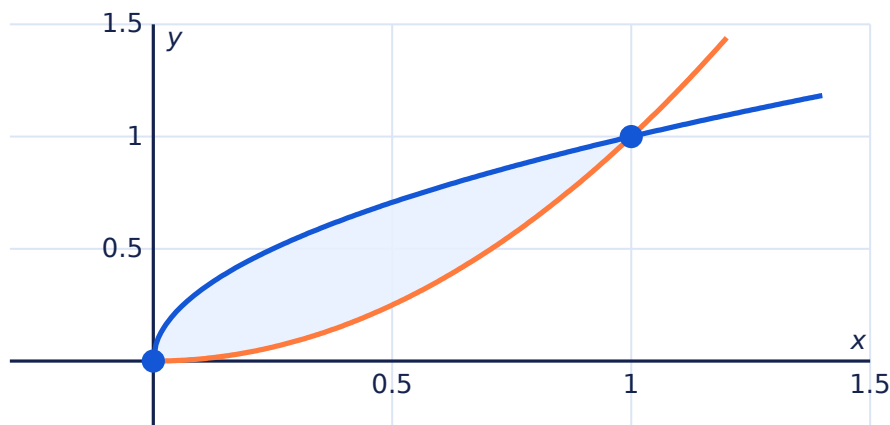
A footbridge model has a base bounded by the x -axis, the curve $y = \sqrt{x}$ and the line $x = 9$. Each vertical slice, cut perpendicular to the x -axis, is a semicircle whose diameter lies across the base. Find the volume of the model.

Problem 9 Disc method (medium)

Find the volume when the region under $y = \sqrt{x+1}$ from $x = 0$ to $x = 3$ is revolved about the x -axis.

Problem 10 Washer method about the x-axis (medium)

Spin the region trapped between $y = \sqrt{x}$ and $y = x^2$ around the x -axis. Find the volume of the solid produced.



The region to spin.

Problem 11 Revolving about a horizontal line (hard)

Rotate the region under $y = x^2$ for $0 \leq x \leq 1$ around the horizontal line $y = -1$ and find the volume swept out.

Problem 12 Revolving about the y-axis (hard)

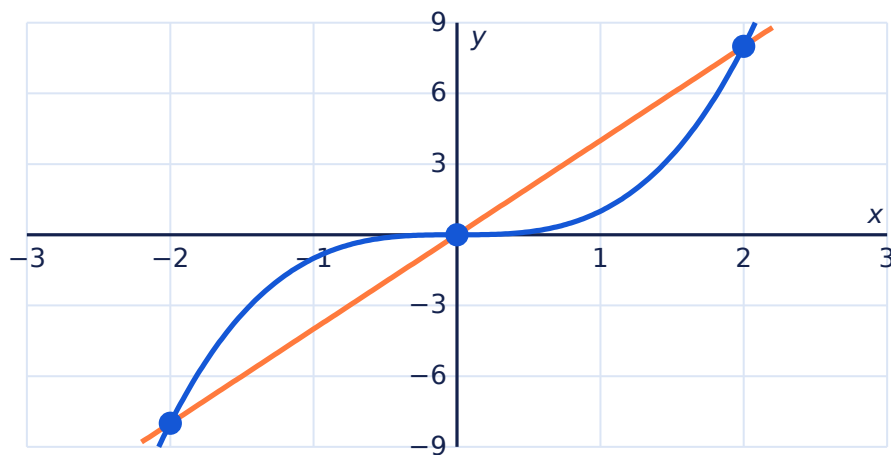
Rotate the area between the parabola $y = x^2$ and the y -axis, below the height $y = 4$, about the y -axis. Integrate in y to get the volume.

Problem 13 Tank with water flowing in and out (hard)

A cistern starts with 120 gallons. For the next 10 minutes the inflow is $2t + 4$ gallons per minute while a drain removes 10 gallons per minute. How much water is in the cistern at $t = 10$, and when is the amount smallest?

Problem 14 Area with three intersection points (hard)

The graphs of $y = x^3$ and $y = 4x$ cross three times. Add the areas of the two bounded pieces.



The cubic and the line.

Problem 15 Area where curves cross (hard)

Find the area of the region between $y = \sin x$ and $y = \cos x$ on $[0, \pi/2]$.

Problem 16 Revolving about a vertical line (hard)

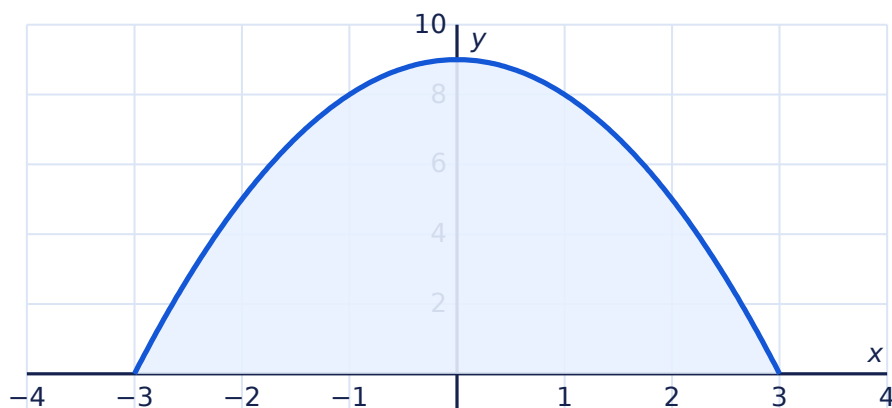
Spin the area under $y = \sqrt{x}$ for $0 \leq x \leq 4$ around the vertical line $x = 4$ and compute the volume by integrating in y .

Problem 17 Greenhouse temperature (hard)

The temperature inside a greenhouse, in °F, is $G(h) = 58 + 12 \sin(\pi h/12)$, where h counts hours from 6 p.m. Find the mean temperature over the 12 hours from $h = 0$ to $h = 12$.

Problem 18 Volume of a tent (hard)

A tent has a base shaped like the region between the x -axis and $y = 9 - x^2$ (feet). Each cross section perpendicular to the x -axis is an isosceles right triangle with a leg in the base. Find the volume.



The tent base.

Problem 19 Area and volume from one region (hard)

A garden bed (in meters) has the shape of the area under $y = 1/x$ between $x = 1$ and $x = e$. Find its area, and the volume of the solid made by spinning the bed around the x -axis.

Problem 20 Mean Value Theorem for integrals (hard)

Find the mean value of $f(x) = x^2 + 1$ over $[1, 4]$, then locate a number c in $[1, 4]$ where $f(c)$ equals that mean.