

Chain Rule and Implicit Differentiation

Grade 12 – AP Calculus AB math · Lesson · Standards: AP Calculus AB Unit 3

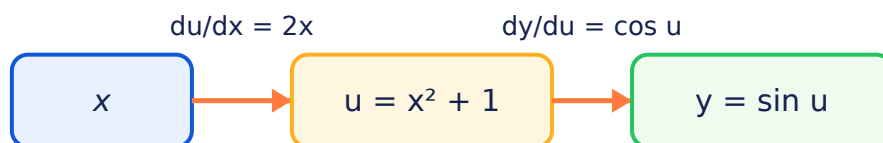
Quick answer: The chain rule finds the derivative of a composite function by multiplying the derivative of the outer function, evaluated at the inner function, by the derivative of the inner function. Implicit differentiation extends this idea to equations where y is not isolated, and inverse formulas handle reflected graphs.

Standards: AP Calculus AB Unit 3 · **Level:** Grade 12 AP Calculus AB

How does the chain rule differentiate a composition?

Chain rule

If $y = f(g(x))$, then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$. Differentiate the outer function, leave the inner function alone, then multiply by the derivative of the inner function.



$$\frac{dy}{dx} = \left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right) = \cos(x^2 + 1) \cdot 2x$$

Each link in the chain contributes one derivative factor.

Worked example

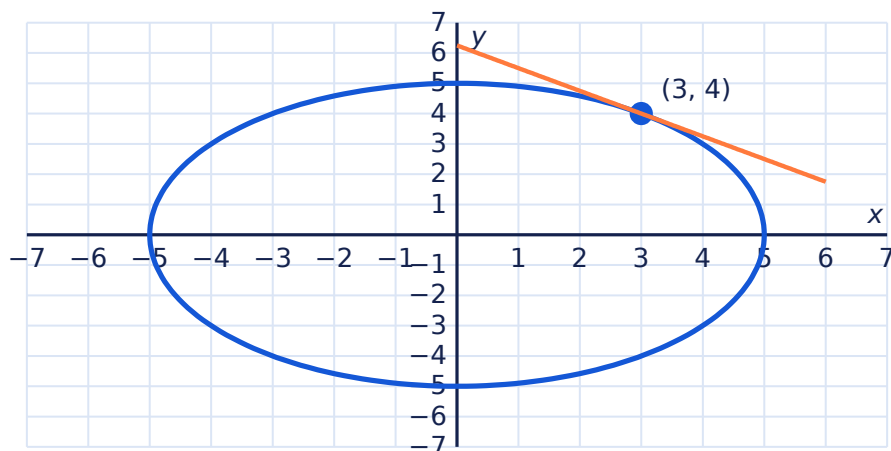
Differentiate $y = \ln(x^2 + 4x)$. The outer function is $\ln u$ with derivative $1/u$. The inner derivative is $2x + 4$. So $\frac{dy}{dx} = \frac{(2x + 4)}{(x^2 + 4x)}$.

How does implicit differentiation work?

Method

1. Differentiate both sides with respect to x , treating y as a function of x .
2. Attach $\frac{dy}{dx}$ whenever you differentiate a term containing y .
3. Collect the $\frac{dy}{dx}$ terms on one side and solve.

For $x^2 + y^2 = 25$, differentiate to get $2x + 2y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{x}{y}$. At $(3, 4)$ the slope is $-3/4$.



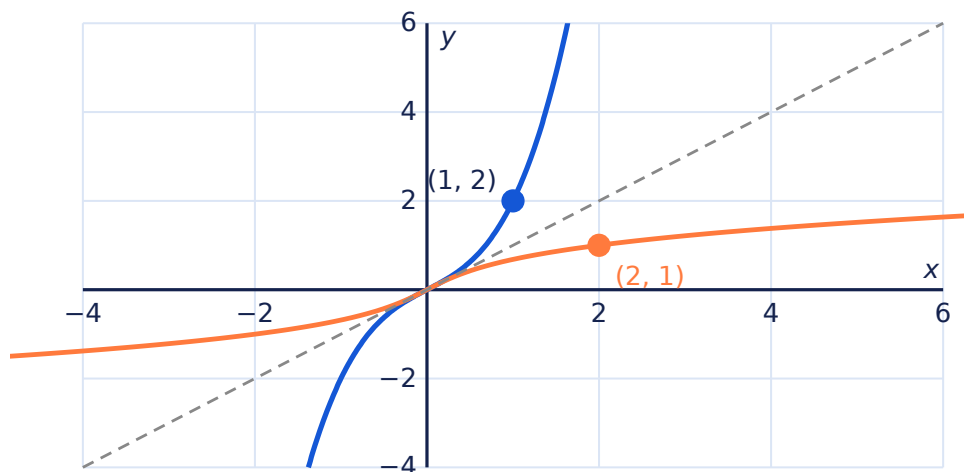
The tangent to the circle at $(3, 4)$ has slope $-3/4$.

What are the derivatives of inverse functions?

Inverse function formulas

If g is the inverse of f , then $g'(b) = 1/f'(g(b))$. The inverse-trigonometric derivatives are:

Function	Derivative
$\arcsin x$	$1/\sqrt{1-x^2}$
$\arccos x$	$-1/\sqrt{1-x^2}$
$\arctan x$	$1/(1+x^2)$



Reflecting across $y = x$ swaps $(1, 2)$ and $(2, 1)$; the slopes become reciprocals, 4 and $1/4$.

Higher-order derivatives repeat differentiation: f'' is the derivative of $f'(x)$, and it measures how fast the slope changes.

Common mistake

Forgetting the inner derivative is the most frequent chain rule error. The derivative of $\sin(3x)$ is

| 3 cos(3x), not cos(3x).
|

Mathbot's tip

Peel the function like an onion: name the outer layer, differentiate it, then move one layer inward and multiply.

Key points to remember

- Chain rule: $d/dx f(g(x)) = f'(g(x)) \cdot g'(x)$.
- Implicit differentiation attaches dy/dx to every term containing y .
- The derivative of an inverse at b is the reciprocal of f' at the matching input.
- Derivatives of arcsin, arccos and arctan come from the inverse function formula.
- The second derivative f'' is the derivative of f' and measures concavity.

Chain Rule and Implicit Differentiation: frequently asked questions

How do I know when to use the chain rule?

Use it when a function sits inside another function, such as $\sin(x^2)$ or $(3x + 1)^5$. If you can name an inner and an outer expression, the chain rule applies.

Why does implicit differentiation include dy/dx ?

Because y depends on x , so differentiating a power of y needs the chain rule. For example, the derivative of y^2 with respect to x is $2y \, dy/dx$.

How do I find a second derivative implicitly?

Differentiate the formula for dy/dx again, using the quotient or product rule. Then substitute the first derivative wherever dy/dx appears and simplify with the original equation.

What does the inverse derivative formula say about slopes?

The tangent slopes of a function and its inverse at matching points are reciprocals. If the slope at $(1, 2)$ is 4, the inverse has slope $1/4$ at $(2, 1)$.

Do I need to memorize the inverse trig derivatives?

Yes. The AP exam expects you to recall the derivatives of arcsin x and arctan x , and then apply the chain rule to inputs such as $3x$ or x^2 .