

Chain Rule and Implicit Differentiation

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 3

Worked solutions with scoring guides follow. Credit method points whenever the chain rule or implicit step is applied correctly, even if arithmetic later goes wrong, but require the final answer in simplified form for full marks.

Standards: AP Calculus AB Unit 3 · **Level:** Grade 12 AP Calculus AB

Question 1 Chain rule evaluation (2 points)

$f(x) = 2^x / (x^2 + 3)$. At $x = 1$: $2/4 = 1/2$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for a correct chain rule setup.

Question 2 Exponential composition (2 points)

$h'(x) = (2x - 4)e^{x^2 - 4x}$. At $x = 3$: $(2)e^{9 - 12} = 2e^{-3}$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for the correct inner derivative.

Question 3 Chain and product from a table (3 points)

(a) $h'(1) = f(g(1)) \cdot g'(1) = f(2) \cdot 3 = (-4)(3) = -12$.

(b) $k'(1) = f(1)g(1) + f(1)g'(1) = 6 \cdot 2 + 5 \cdot 3 = 27$.

(c) $m'(1) = g'(g(1)) \cdot g'(1) = g'(2) \cdot 3 = 2 \cdot 3 = 6$.

Scoring: 1 point per part.

Question 4 Tangent line to an implicit curve (3 points)

(a) Differentiate: $y^2 + 2xy \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} = 0$. Solve: $\frac{dy}{dx} = \frac{-(y^2 + 2xy)}{(2xy + x^2)}$.

(b) At $(1, 2)$: $-(4 + 4)/(4 + 1) = -8/5$. The line is $y - 2 = -8/5(x - 1)$.

Scoring: Part (a): 1 point for differentiating both terms correctly, 1 point for solving. Part (b): 1 point.

Question 5 Inverse function derivative (3 points)

(a) Since $f(1) = 1 + 4 + 3 = 8$, $g(8) = 1$. Also $f(x) = 3x^2 + 4$, so $f'(1) = 7$ and $g'(8) = \mathbf{1/7}$.

(b) The derivative of $\arctan u$ is $u'/(1 + u^2)$: $\frac{2x}{(1 + x^4)}$.

Scoring: Part (a): 1 point for locating the input 1, 1 point for the reciprocal. Part (b): 1 point.

Question 6 First and second derivatives (3 points)

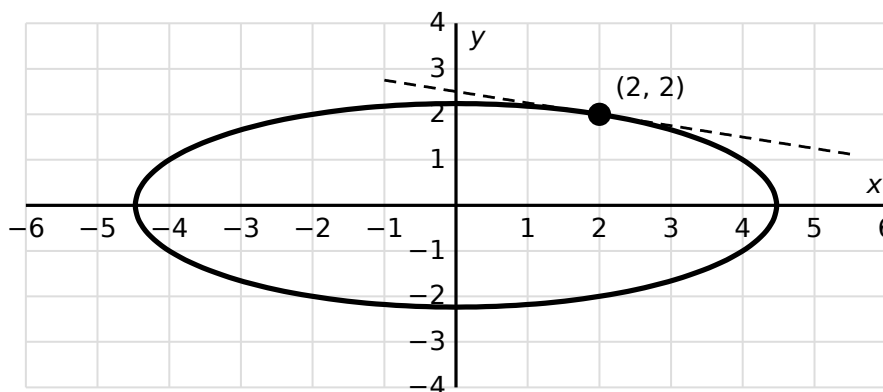
(a) Product rule: $f(x) = 2xe^{-x} - x^2e^{-x} = \mathbf{(2x - x^2)e^{-x}}$.

(b) Differentiate again: $(2 - 2x)e^{-x} - (2x - x^2)e^{-x} = \mathbf{(x^2 - 4x + 2)e^{-x}}$.

Scoring: Part (a): 1 point. Part (b): 1 point for the product rule setup, 1 point for the simplification.

Question 7 Slope on an ellipse (2 points)

Differentiate: $2x + 8y \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{x}{4y}$. At $(2, 2)$: $-2/8 = \mathbf{-1/4}$.



The tangent at $(2, 2)$ has slope $-1/4$.

Scoring: 1 point for differentiating implicitly, 1 point for the correct slope.

Question 8 Derivative of a squared sine (2 points)

Chain rule: $2 \sin(3x) \cdot \cos(3x) \cdot 3 = 6 \sin(3x) \cos(3x)$. Since $2 \sin \theta \cos \theta = \sin 2\theta$, this equals $\mathbf{3 \sin(6x)}$.

Scoring: 1 point for the chain rule, 1 point for the simplification.