

Chain Rule and Implicit Differentiation

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 3

Name: _____ Date: _____ Class: _____

This 45-minute test mixes written multiple-choice items and free-response questions in the AP Calculus AB style. Show all work for free-response items. The total is 20 points, and no calculator is needed unless a question says otherwise.

Standards: AP Calculus AB Unit 3 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

Question 1 Chain rule evaluation (2 points)

If $f(x) = \ln(x^2 + 3)$, what is $f'(1)$?

- A. $1/4$
- B. $1/2$
- C. 2
- D. $1/3$

Question 2 Exponential composition (2 points)

If $h(x) = e^{x^2 - 4x}$, what is $h'(3)$?

- A. e^{-3}
- B. $2e^{-3}$
- C. 2
- D. $6e^{-3}$

Question 3 Chain and product from a table (3 points)

Values of f , g and their derivatives are given.

x	1	2
$f(x)$	5	7
$f'(x)$	6	-4
$g(x)$	2	1

x	1	2
$g'(x)$	3	2

- If $h(x) = f(g(x))$, find $h'(1)$. (1 point)
- If $k(x) = f(x)g(x)$, find $k'(1)$. (1 point)
- If $m(x) = g(g(x))$, find $m'(1)$. (1 point)

Question 4 Tangent line to an implicit curve (3 points)

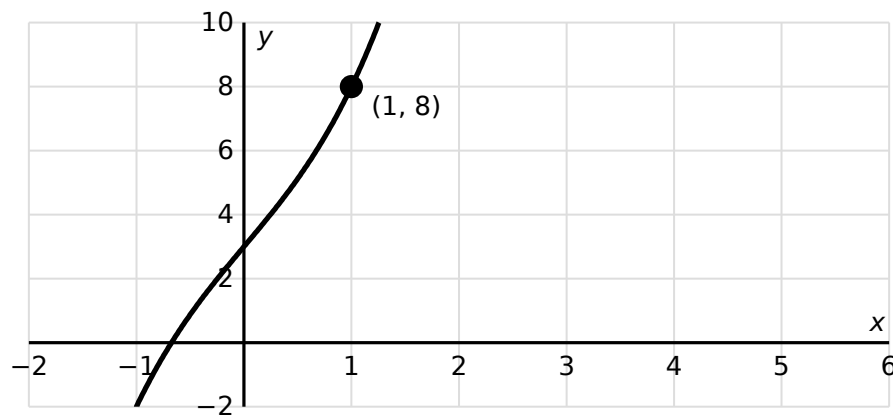
The curve $xy^2 + x^2y = 6$ passes through (1, 2).

- Find $\frac{dy}{dx}$ in terms of x and y . (2 points)
- Write an equation of the tangent line at (1, 2). (1 point)

Question 5 Inverse function derivative (3 points)

Let $f(x) = x^3 + 4x + 3$, which is increasing, and let g be its inverse.

- Find $g'(8)$. (2 points)
- Find the derivative of $\arctan(x^2)$. (1 point)



Graph of f .

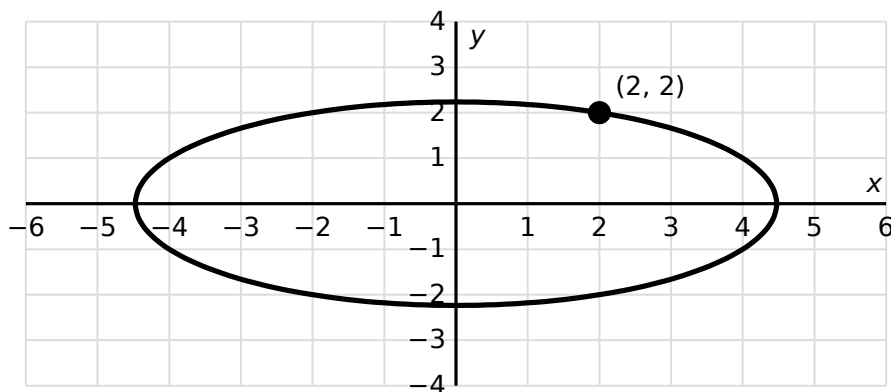
Question 6 First and second derivatives (3 points)

Let $f(x) = x^2e^{-x}$.

- Find $f'(x)$. (1 point)
- Find $f''(x)$. (2 points)

Question 7 Slope on an ellipse (2 points)

The ellipse $x^2 + 4y^2 = 20$ is shown. Find the slope of the tangent line at $(2, 2)$.



Graph of the ellipse.

Question 8 Derivative of a squared sine (2 points)

Find the derivative of $f(x) = \sin^2(3x)$ and write it in the form $a \sin(bx)$.