

Chain Rule and Implicit Differentiation

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 3

Name: _____ Date: _____ Class: _____

These 20 problems follow AP Calculus AB Unit 3: the chain rule, implicit differentiation, derivatives of inverse functions and inverse trigonometric functions, and higher-order derivatives. Early problems isolate one layer of a composition; later ones combine several rules.

The final six problems mix contexts and multi-step reasoning, including a pressure model, a camera angle and a curve with horizontal and vertical tangents.

Standards: AP Calculus AB Unit 3 · **Level:** Grade 12 AP Calculus AB

Problem 1 Chain rule with a power (easy)

Differentiate $f(x) = (6x + 1)^4$ with respect to x .

Problem 2 Chain rule with a root (easy)

Compute the derivative of $f(x) = \sqrt{3x^2 + 16}$.

Problem 3 Chain rule with e (easy)

Find $f'(x)$ for $f(x) = e^{x^2 - 3x}$.

Problem 4 Chain rule with a logarithm (easy)

Find $f'(x)$ for $f(x) = \ln(x^2 + 4x)$.

Problem 5 Chain rule with sine (easy)

Find $f'(x)$ for $f(x) = \sin(3x^2)$.

Problem 6 Chain rule at a point (medium)

Let $f(x) = (2x^2 - 1)^3$. Evaluate $f'(1)$.

Problem 7 Chain rule from a table (medium)

Let $h(x) = f(g(x))$. Use the table to find $h'(1)$.

x	1	2
$g(x)$	2	1
$g'(x)$	4	2
$f(x)$	5	7
$f'(x)$	6	-3

Problem 8 Product rule with the chain rule (medium)

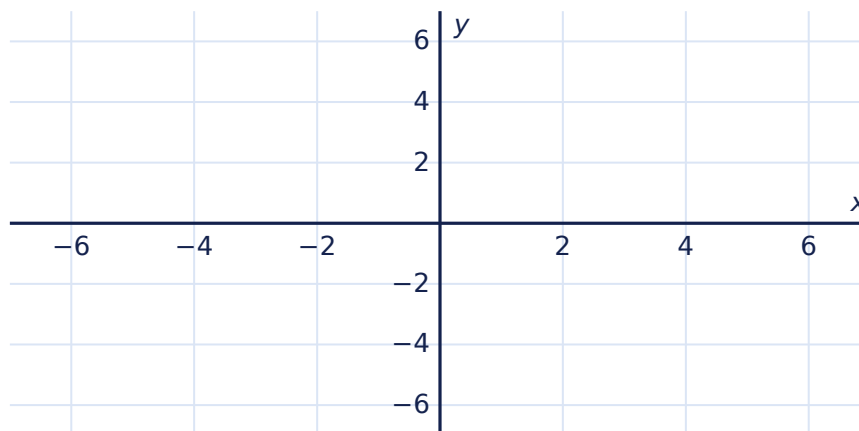
Use the product rule together with the chain rule to differentiate $f(x) = x^3 e^{-2x}$.

Problem 9 Quotient rule with the chain rule (medium)

Find $f'(x)$ for $f(x) = \frac{(\sin 2x)}{x^2}$.

Problem 10 Slope on a circle (medium)

The circle $x^2 + y^2 = 169$ contains the point $(5, 12)$. Find $\frac{dy}{dx}$ and the slope of the tangent line at $(5, 12)$.



Circle of radius 13.

Problem 11 Implicit differentiation with a product (medium)

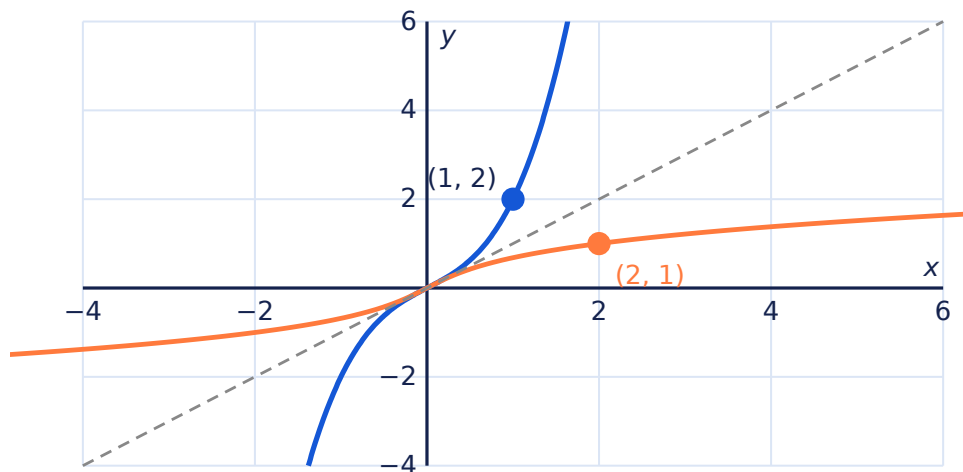
The curve $x^2 y + y^3 = 10$ contains $(1, 2)$. Find $\frac{dy}{dx}$ at that point.

Problem 12 Tangent line to an implicit curve (medium)

Write the equation of the line tangent to $x^2 + xy + y^2 = 7$ at $(2, 1)$.

Problem 13 Derivative of an inverse function (medium)

Let $f(x) = x^3 + x$, which is one-to-one, and let g be its inverse. Find $g'(10)$.



Graph of f and its inverse g .

Problem 14 Inverse derivative from a table (medium)

The function f is differentiable and one-to-one with the values shown. If g is its inverse, find $g'(7)$.

x	1	2	3
$f(x)$	4	7	11
$f'(x)$	2	3	5

Problem 15 Inverse trigonometric derivatives (hard)

Find the derivative of

- a. $\arctan(4x)$
- b. $\arcsin(x^2)$

Problem 16 Second derivative of a sum (hard)

Let $f(x) = x^5 - 4x^3 + e^{2x}$. Find $f''(x)$ and $f''(0)$.

Problem 17 Second derivative of an implicit curve (hard)

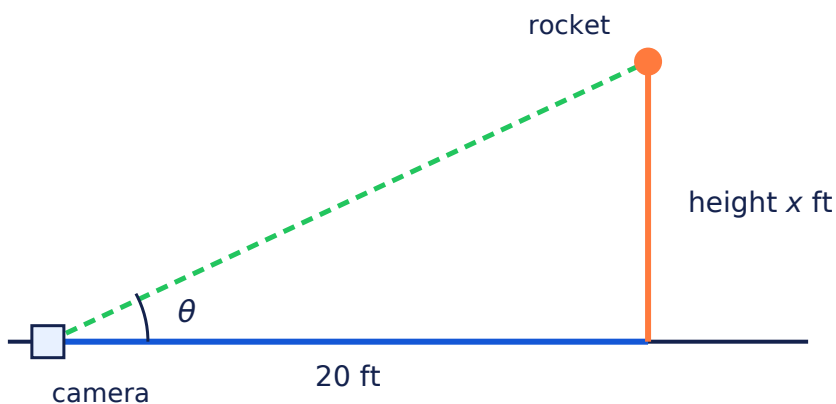
On the circle $x^2 + y^2 = 36$, find $\frac{d^2y}{dx^2}$ at the point $(0, 6)$.

Problem 18 Pressure on a climbing balloon (hard)

Air pressure, in kilopascals, at height h kilometers is $P(h) = 100e^{-0.2h}$. A balloon's height is $h(t) = 5 + 3t$ km after t hours. Find the rate of change of pressure at $t = 2$.

Problem 19 Camera tracking a rocket (hard)

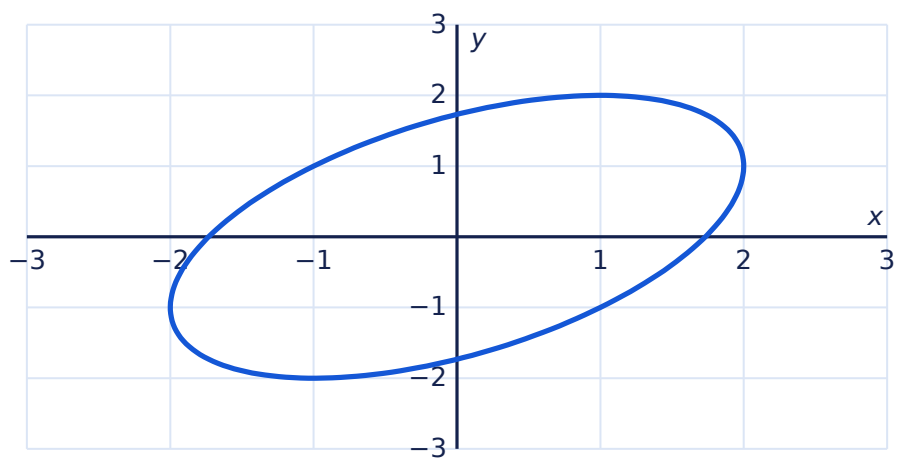
A camera sits 20 feet from the base of a launch pad and tracks a rocket at height x feet. The angle of elevation is $\theta = \arctan(x/20)$. Find $\frac{d\theta}{dx}$ when $x = 15$.



Camera and rocket.

Problem 20 Horizontal and vertical tangents (hard)

Find all points on $x^2 - xy + y^2 = 3$ where the tangent line is horizontal and all points where it is vertical.



The curve $x^2 - xy + y^2 = 3$.