

Derivative Definition and Differentiation Rules

Grade 12 – AP Calculus AB math · Lesson · Standards: AP Calculus AB Unit 2

Quick answer: The derivative measures how fast a function changes at a single input. It is defined as the limit of the difference quotient, equals the slope of the tangent line, and is computed quickly with the power, product, quotient and trigonometric rules covered in AP Calculus AB Unit 2.

Standards: AP Calculus AB Unit 2 · **Level:** Grade 12 AP Calculus AB

What is the derivative of a function?

Derivative at a point

The derivative of f at $x = a$ is the limit of the slopes of secant lines: $f'(a) = \lim_{h \rightarrow 0} \frac{(f(a+h) - f(a))}{h}$. It is the instantaneous rate of change and the slope of the tangent line.



Secant slope from $x = 2$ to $x = 4$ is 3. As the second point slides toward P, the secant approaches the tangent of slope 2.

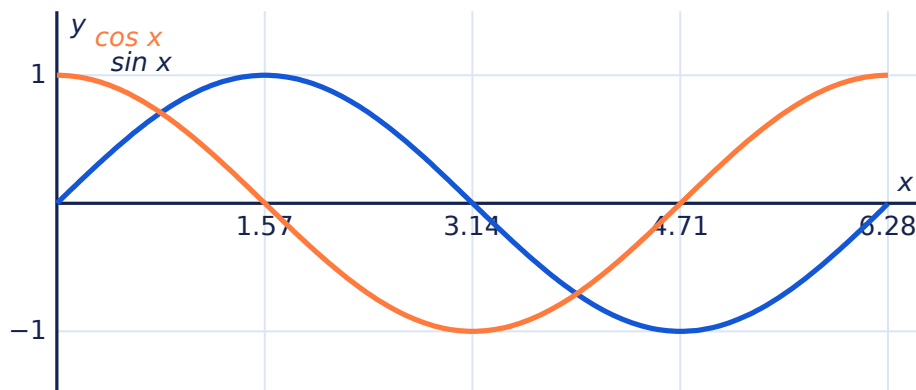
Using the definition

For $f(x) = 3x^2 - x$ at $x = 2$: $\frac{(3(2+h)^2 - (2+h) - 10)}{h} = \frac{(11h + 3h^2)}{h} = 11 + 3h$. As $h \rightarrow 0$ the result is **11**.

Which rules give derivatives quickly?

Function	Derivative
x^n	nx^{n-1}
$c \cdot f(x)$ and $f(x) + g(x)$	$c \cdot f'(x)$ and $f'(x) + g'(x)$

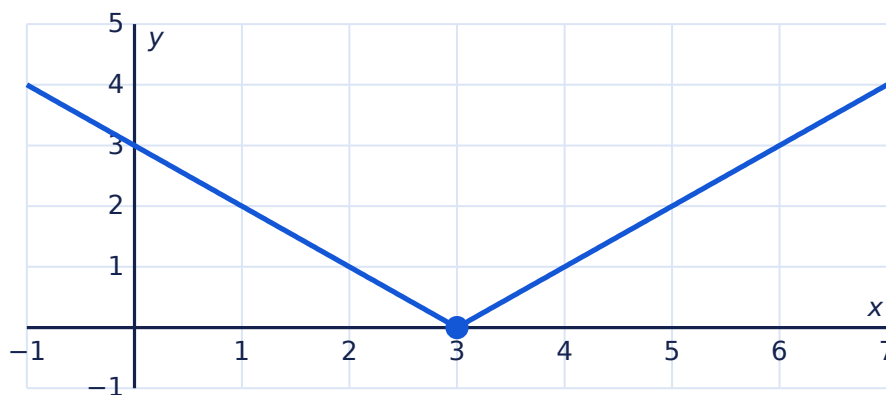
Function	Derivative
$e^x, \ln x$	$e^x, 1/x$
$\sin x, \cos x$	$\cos x, -\sin x$
$\tan x, \sec x$	$\sec^2 x, \sec x \tan x$
$\cot x, \csc x$	$-\csc^2 x, -\csc x \cot x$
Product $f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
Quotient $f(x)/g(x)$	$(f'(x)g(x) - f(x)g'(x))/g(x)^2$



The derivative of sine is cosine: where the blue curve peaks, the orange curve crosses zero.

When does a derivative fail to exist?

A function is not differentiable at a point of discontinuity, at a corner, at a cusp, or where the tangent line is vertical. Differentiable always implies continuous, but continuous does not imply differentiable.



The graph of $|x - 3|$ is continuous at 3 but has slope -1 on the left and $+1$ on the right, so $f'(3)$ does not exist.

Method for a product

1. Name the two factors and find each derivative separately.
2. Apply first·(derivative of second) + second·(derivative of first).
3. Simplify by expanding and combining like terms.

Common mistake

The derivative of a product is not the product of the derivatives, and the quotient rule subtracts in a fixed order: bottom times derivative of top, minus top times derivative of bottom.

Mathbot's tip

Say the rule out loud while you write it. "Derivative of the first, times the second, plus the first, times derivative of the second" keeps both terms in the answer.

Key points to remember

- The derivative is the limit as h approaches 0 of the difference quotient $(f(a+h) - f(a))/h$.
- Differentiable at a point implies continuous there, but not the reverse.
- The power rule gives d/dx of x^n as $n \cdot x^{(n-1)}$; $\sin$, $\cos$, e^x and $\ln x$ have their own derivatives.
- The product rule has two terms, and the quotient rule divides by the square of the denominator.
- Table values estimate a derivative with the slope of a nearby secant line.

Derivative Definition and Differentiation Rules: frequently asked questions

What is the difference between average and instantaneous rate of change?

The average rate is the slope of a secant line over an interval. The instantaneous rate is the limit of those slopes as the interval shrinks to a point, which is the derivative.

Can a function be continuous but not differentiable?

Yes. The graph of $|x|$ is continuous everywhere, but it has a corner at 0, so the slopes from the left and right differ and no derivative exists.

When do I use the product rule instead of the power rule?

Use the product rule when two non-constant expressions in x are multiplied, such as $x^2 \sin x$. A constant times a function only needs the constant multiple rule.

How do I estimate a derivative from a table?

Find the slope of the secant line between the two table values closest to the input, on either side if possible. The result approximates the derivative with units of output per input.

What is the derivative of $\ln x$ and why does it matter?

The derivative of $\ln x$ is $1/x$. It appears in quotient and product problems and in tangent-line questions where the point of tangency is unknown.