

# Derivative Definition and Differentiation Rules

## Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 2

The worked solutions below carry a scoring guide. Give credit for correct setup even when a later arithmetic slip changes the final number, but require the correct rule to be named or clearly used.

**Standards:** AP Calculus AB Unit 2 · **Level:** Grade 12 AP Calculus AB

### Question 1 Product rule evaluation (2 points)

$f(x) = 3x^2e^x + x^3e^x = (x^3 + 3x^2)e^x$ . So  $f(1) = 4e$ . Answer **(B)**.

*Scoring: 2 points for the correct choice; 1 point for a correct application of the product rule.*

### Question 2 Quotient rule evaluation (2 points)

$f(x) = \frac{(x \cos x - \sin x)}{x^2}$ . At  $x = \pi$ :  $(\pi(-1) - 0)/\pi^2 = -1/\pi$ . Answer **(A)**.

*Scoring: 2 points for the correct choice; 1 point for a correct quotient rule setup.*

### Question 3 Limit definition (3 points)

$f(5) = 75 - 10 = 65$ . The difference quotient is  $\frac{(3(5+h)^2 - 2(5+h) - 65)}{h} = \frac{(28h + 3h^2)}{h} = 28 + 3h$ . As  $h \rightarrow 0$ :  $f(5) = \mathbf{28}$ .

*Scoring: 1 point for the correct difference quotient, 1 point for expanding and simplifying, 1 point for the limit.*

### Question 4 Snow depth table (3 points)

(a)  $(25 - 4)/(7 - 0) = \mathbf{3 \text{ cm/h}}$ .

(b) Use the points at 3 and 7:  $(25 - 11)/(7 - 3) = 14/4 = \mathbf{3.5 \text{ cm/h}}$ .

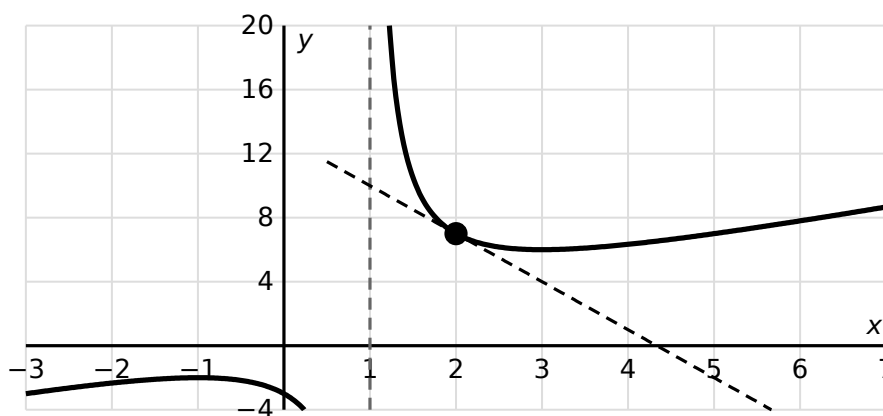
(c) At 5 hours after the storm began, the snow depth is increasing at about 3.5 centimeters per hour.

*Scoring: 1 point per part; units required in (b) or (c).*

### Question 5 Tangent line to a rational function (3 points)

(a) Quotient rule:  $f'(x) = \frac{(2x(x-1) - (x^2+3))}{(x-1)^2} = \frac{(x^2 - 2x - 3)}{(x-1)^2}$ .

(b)  $f(2) = 7$  and  $f'(2) = (4 - 4 - 3)/1 = -3$ . The line is  $y - 7 = -3(x - 2)$ , or  **$y = -3x + 13$** .



The tangent line at (2, 7) has slope  $-3$ .

Scoring: Part (a): 1 point for the setup, 1 for the simplification. Part (b): 1 point.

### Question 6 Piecewise differentiability (3 points)

(a) Left value  $4a + 1$  equals right value  $6 + b$ .

(b) The left derivative is  $2ax = 4a$  at  $x = 2$ , and the right derivative is 3. So  $4a = 3$ .

(c)  $a = \mathbf{3/4}$ . Then  $3 + 1 = 6 + b$ , so  $b = \mathbf{-2}$ .

Scoring: 1 point per part.

### Question 7 Where the derivative fails (2 points)

At  $x = 1$  the graph has a corner, with slope  $-1$  on the left and  $+1$  on the right. At  $x = 3$  the graph has a jump (the function is discontinuous), so it cannot be differentiable.  **$x = 1$  and  $x = 3$** .

Scoring: 1 point for each correct value with its reason.

### Question 8 Trigonometric derivatives (2 points)

The derivative of  $\tan x$  is  $\sec^2 x$ . The derivative of  $\csc x$  is  $-\csc x \cot x$ . So  **$f'(x) = 4 \sec^2 x + \csc x \cot x$** .

Scoring: 1 point per term.