

Derivative Definition and Differentiation Rules

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 2

Name: _____ Date: _____ Class: _____

This 45-minute test mixes written multiple-choice items with free-response questions in the AP Calculus AB style. Show all work, and give units where the context requires them. The total is 20 points. No calculator is needed.

Standards: AP Calculus AB Unit 2 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

Question 1 Product rule evaluation (2 points)

If $f(x) = x^3 e^x$, what is $f'(1)$?

- A. $3e$
- B. $4e$
- C. $e + 3$
- D. 4

Question 2 Quotient rule evaluation (2 points)

If $f(x) = \frac{(\sin x)}{x}$, what is $f'(\pi)$?

- A. $-1/\pi$
- B. $1/\pi$
- C. $-\pi$
- D. 0

Question 3 Limit definition (3 points)

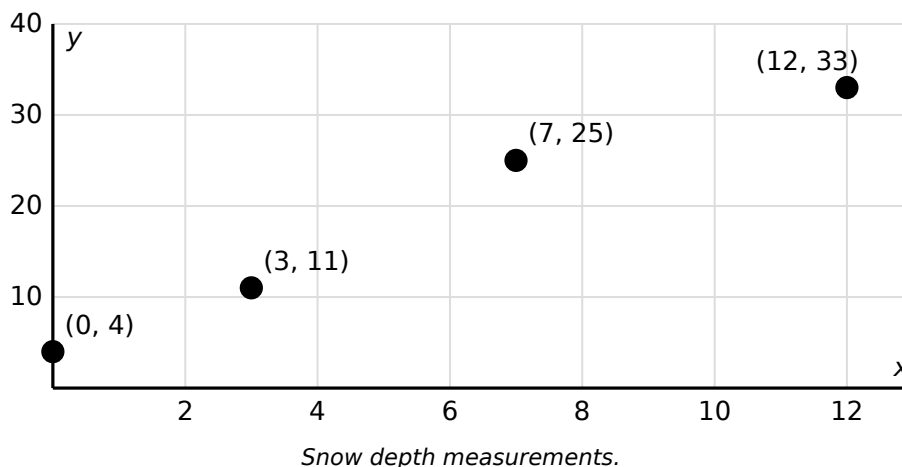
Use the limit definition of the derivative to find $f'(5)$ for $f(x) = 3x^2 - 2x$. Show every step.

Question 4 Snow depth table (3 points)

The depth $D(t)$ of snow, in centimeters, t hours after a storm begins is plotted.

- a. Find the average rate of change of D on $[0, 7]$. (1 point)

- b. Use the nearest data points to estimate $D'(5)$. (1 point)
 c. Interpret your answer to (b) with units. (1 point)



Question 5 Tangent line to a rational function (3 points)

Let $f(x) = \frac{(x^2 + 3)}{(x - 1)}$.

- a. Find $f'(x)$. (2 points)
 b. Write an equation of the tangent line at $x = 2$. (1 point)

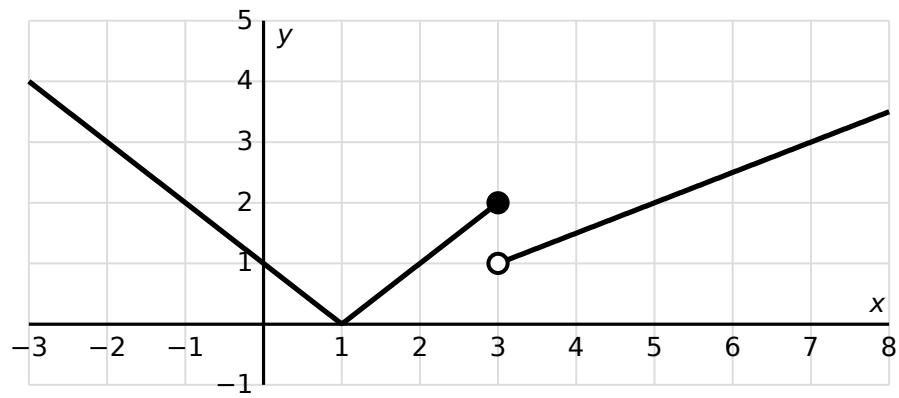
Question 6 Piecewise differentiability (3 points)

Let $f(x) = ax^2 + 1$ for $x \leq 2$, and $f(x) = 3x + b$ for $x > 2$. The function is differentiable at $x = 2$.

- a. Write the equation that makes f continuous at 2. (1 point)
 b. Write the equation that makes the derivatives equal at 2. (1 point)
 c. Find a and b . (1 point)

Question 7 Where the derivative fails (2 points)

The graph of f is shown on $[-3, 8]$. List every x -value in $(-3, 8)$ where $f'(x)$ does not exist, and give the reason for each.



Graph of f .

Question 8 Trigonometric derivatives (2 points)

Find $f'(x)$ for $f(x) = 4 \tan x - \csc x$.