

Derivative Definition and Differentiation Rules

Grade 12 – AP Calculus AB math · Worksheet Answer Key · Standards: AP Calculus AB Unit 2

Each solution shows the rule used and the algebra after it. If your result differs, check the sign of each term, the order of terms in the quotient rule, and the units in context problems.

Standards: AP Calculus AB Unit 2 · **Level:** Grade 12 AP Calculus AB

Problem 1 Average rate of change

$f(4) = 64 - 20 = 44$ and $f(1) = 1 - 5 = -4$. The average rate is $\frac{(44 - (-4))}{(4 - 1)} = 48/3 = \mathbf{16}$.

Problem 2 Rate of change from a table

$\frac{(1,630 - 1,310)}{(6 - 2)} = 320/4 = \mathbf{80 \text{ feet per mile}}$.

Problem 3 Power rule with a polynomial

Differentiate each term: $7 \cdot 4x^3 - 5 \cdot 2x + 0$. So $f'(x) = \mathbf{28x^3 - 10x}$.

Problem 4 Power rule with roots and fractions

Rewrite as $10x^{1/2} - 12x^{-4}$. Then $f'(x) = 5x^{-1/2} + 48x^{-5} = \frac{5}{\sqrt{x}} + \frac{48}{x^5}$.

Problem 5 Derivatives of sine and cosine

The derivative of $\sin x$ is $\cos x$, and the derivative of $\cos x$ is $-\sin x$. So $f'(x) = 5 \cos x - 2(-\sin x) = \mathbf{5 \cos x + 2 \sin x}$.

Problem 6 Exponential and logarithm terms

Differentiate term by term: $3e^x + 4/x - 1$. So $f'(x) = \mathbf{3e^x + \frac{4}{x} - 1}$.

Problem 7 Limit definition at a point

$f(5) = 25 + 35 = 60$. Then $\frac{((5 + h)^2 + 7(5 + h) - 60)}{h} = \frac{(17h + h^2)}{h} = 17 + h$. As $h \rightarrow 0$ the limit is $\mathbf{17}$.

Problem 8 Product rule

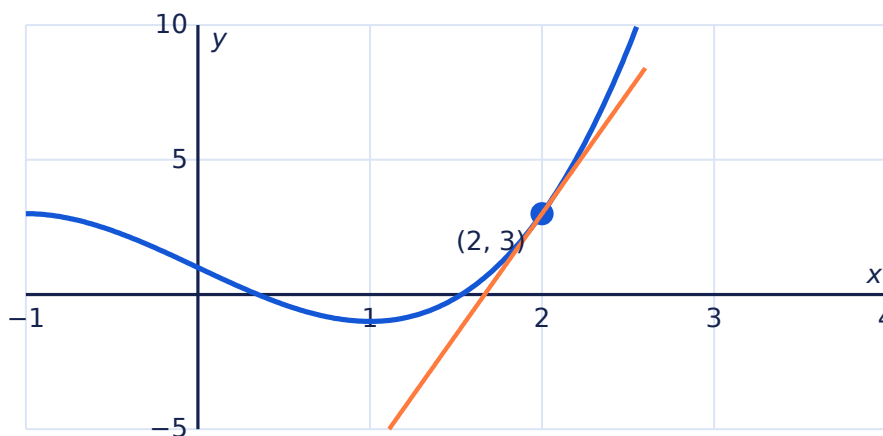
Product rule: $6x(x^3 - 4x) + (3x^2 + 1)(3x^2 - 4)$. Expand: $6x^4 - 24x^2 + 9x^4 - 12x^2 + 3x^2 - 4$.
Combine: $f(x) = 15x^4 - 33x^2 - 4$.

Problem 9 Quotient rule

$f(x) = \frac{(2(x^2 + 1) - (2x + 5)(2x))}{((x^2 + 1)^2)}$. The numerator is $2x^2 + 2 - 4x^2 - 10x = -2x^2 - 10x + 2$. So $f(x) = \frac{(-2x^2 - 10x + 2)}{((x^2 + 1)^2)}$.

Problem 10 Equation of a tangent line

$f(2) = 8 - 6 + 1 = 3$. Since $f(x) = 3x^2 - 3$, $f(2) = 12 - 3 = 9$. Point-slope form: $y - 3 = 9(x - 2)$, so $y = 9x - 15$.



The tangent line touches the curve at (2, 3).

Problem 11 Derivatives of tangent and secant

The derivative of $\tan x$ is $\sec^2 x$ and the derivative of $\sec x$ is $\sec x \tan x$. So $f(x) = \sec^2 x - 2 \sec x \tan x$.

Problem 12 Product rule with a trig factor

$f(x) = 2x \sin x + x^2 \cos x$. At $x = \pi$: $2\pi(0) + \pi^2(-1) = -\pi^2$.

Problem 13 Quotient rule with a logarithm

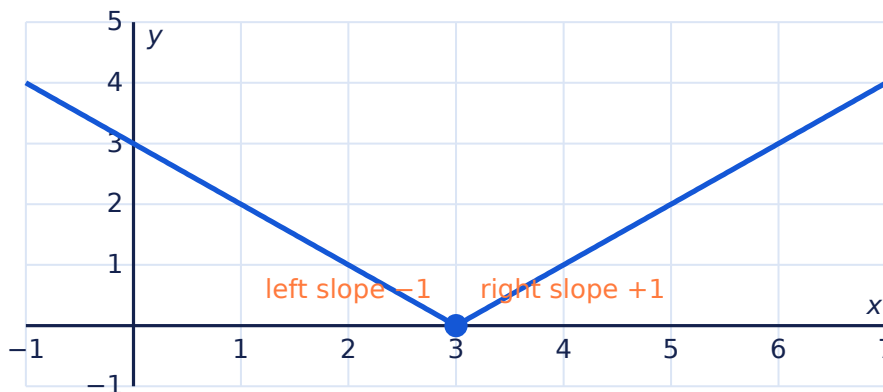
$f(x) = \frac{((1/x)x^2 - (\ln x)(2x))}{x^4} = \frac{(x - 2x \ln x)}{x^4} = \frac{(1 - 2 \ln x)}{x^3}$. At $x = e$, $\ln e = 1$, so $f(e) = \frac{(-1)}{e^3}$.

Problem 14 Differentiable piecewise function

Continuity: the left value is 9, so $3a + b = 9$. Equal slopes: the left derivative is $2x = 6$, so $a = 6$. Then $b = 9 - 18 = -9$. The values are **$a = 6$ and $b = -9$** .

Problem 15 Checking differentiability at a corner

For $x < 3$, $f(x) = 3 - x$, with slope -1 . For $x > 3$, $f(x) = x - 3$, with slope $+1$. The one-sided derivatives disagree, so **$f(3)$ does not exist** even though f is continuous at 3.



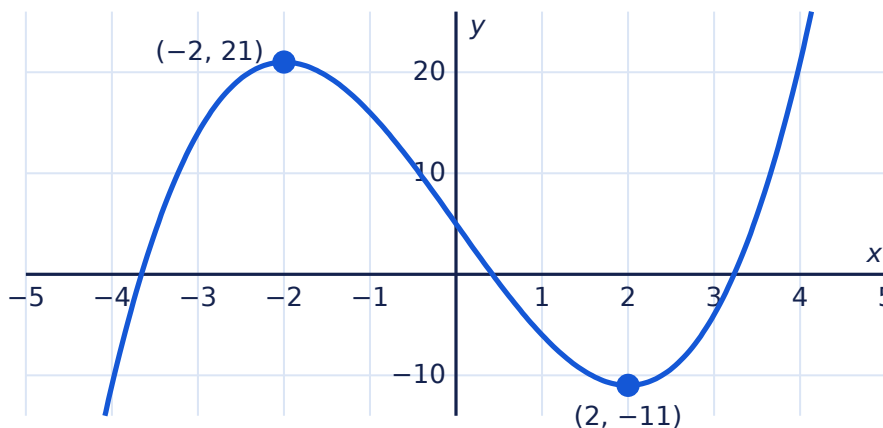
The slopes -1 and $+1$ meet at the corner.

Problem 16 Estimating a derivative from a table

Use the values on either side of 4: $\frac{D(6) - D(2)}{(6 - 2)} = \frac{(44 - 18)}{4} = 26/4 = 6.5$ **feet per hour**. The river is rising at about 6.5 ft/h at $t = 4$.

Problem 17 Horizontal tangent lines

Set $f(x) = 3x^2 - 12 = 0$, so $x^2 = 4$ and $x = \pm 2$. Then $f(2) = 8 - 24 + 5 = -11$ and $f(-2) = -8 + 24 + 5 = 21$. The points are **$(2, -11)$ and $(-2, 21)$** .



Problem 18 Drone height

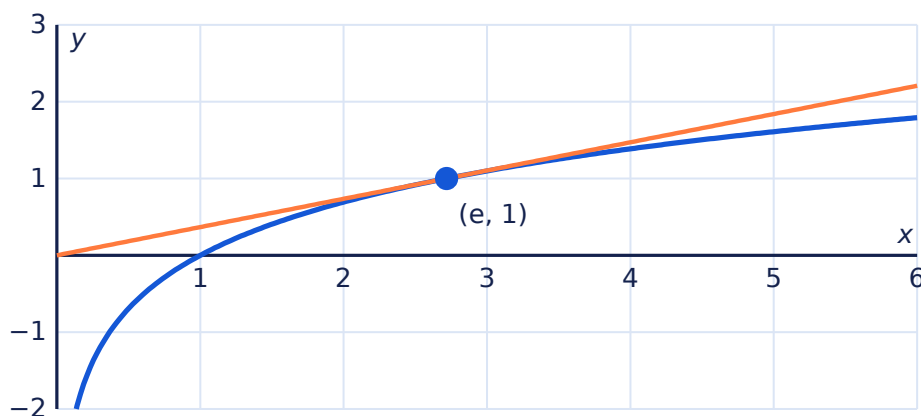
$h(1) = 32$ and $h(3) = 72$, so the average velocity is $40/2 = \mathbf{20 \text{ ft/s}}$. The derivative is $h'(t) = -8t + 36$, so $h'(2) = \mathbf{20 \text{ ft/s}}$. Setting $-8t + 36 = 0$ gives $t = \mathbf{4.5 \text{ s}}$, when the drone reaches its top height $h(4.5) = 81 \text{ ft}$.

Problem 19 Concentration of a medicine

Quotient rule: $C'(t) = \frac{(8(t^2 + 4) - 8t(2t))}{(t^2 + 4)^2} = \frac{(32 - 8t^2)}{(t^2 + 4)^2}$. Then $C'(1) = 24/25 = \mathbf{0.96 \text{ mg/L per hour}}$. The rate is zero when $32 = 8t^2$, so $t = \mathbf{2 \text{ hours}}$, when $C(2) = 2 \text{ mg/L}$.

Problem 20 Tangent line through the origin

Let the point of tangency be $(a, \ln a)$ with slope $1/a$. The tangent line is $y - \ln a = (1/a)(x - a)$. Substituting $(0, 0)$: $-\ln a = -1$, so $\ln a = 1$ and $a = e$. The slope is $1/e$. The point is $(\mathbf{e}, \mathbf{1})$ and the line is $\mathbf{y = x/e}$.



The tangent line passes through the origin.