

Derivative Definition and Differentiation Rules

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 2

Name: _____ Date: _____ Class: _____

These 20 problems cover AP Calculus AB Unit 2: average and instantaneous rates of change, the limit definition, differentiability, and the rules for powers, products, quotients and trigonometric functions. The first problems are single-step; later ones need several rules at once.

The last six problems are multi-step contexts such as a drone flight and a drug concentration. Show the derivative first, then evaluate or solve.

Standards: AP Calculus AB Unit 2 · **Level:** Grade 12 AP Calculus AB

Problem 1 Average rate of change (easy)

Find the average rate of change of $f(x) = x^3 - 5x$ on the interval $[1, 4]$.

Problem 2 Rate of change from a table (easy)

The elevation E (feet) of a hiking trail at mile m is shown. Find the average rate of change from mile 2 to mile 6 and give units.

m (miles)	0	2	4	6
$E(m)$	1,150	1,310	1,420	1,630

Problem 3 Power rule with a polynomial (easy)

Find $f'(x)$ for $f(x) = 7x^4 - 5x^2 + 9$.

Problem 4 Power rule with roots and fractions (easy)

Find $f'(x)$ for $f(x) = 10\sqrt{x} - \frac{12}{x^4}$.

Problem 5 Derivatives of sine and cosine (easy)

Find $f'(x)$ for $f(x) = 5 \sin x - 2 \cos x$.

Problem 6 Exponential and logarithm terms (easy)

Find $f'(x)$ for $f(x) = 3e^x + 4 \ln x - x$.

Problem 7 Limit definition at a point (medium)

Apply the difference quotient with $h \rightarrow 0$ to compute $f'(5)$ when $f(x) = x^2 + 7x$.

Problem 8 Product rule (medium)

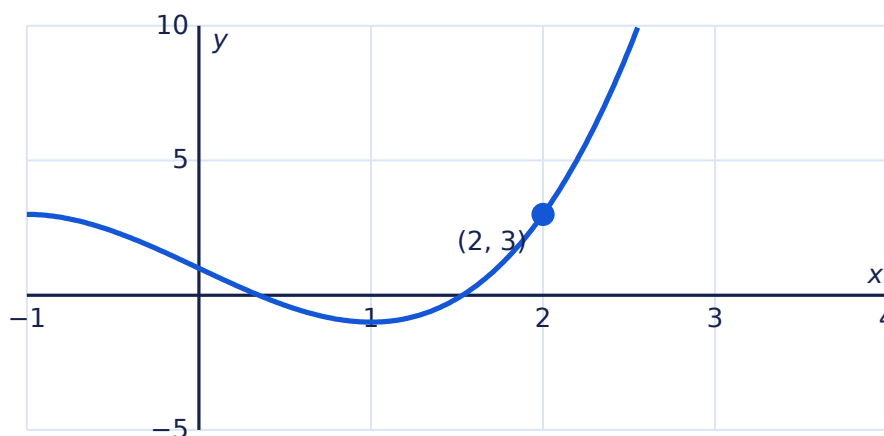
Find $f'(x)$ for $f(x) = (3x^2 + 1)(x^3 - 4x)$.

Problem 9 Quotient rule (medium)

Find $f'(x)$ for $f(x) = \frac{(2x + 5)}{(x^2 + 1)}$.

Problem 10 Equation of a tangent line (medium)

Write an equation of the line tangent to $f(x) = x^3 - 3x + 1$ at $x = 2$.



Graph of f .

Problem 11 Derivatives of tangent and secant (medium)

Find $f'(x)$ for $f(x) = \tan x - 2 \sec x$.

Problem 12 Product rule with a trig factor (medium)

Let $f(x) = x^2 \sin x$. Find $f'(x)$ and evaluate $f'(\pi)$.

Problem 13 Quotient rule with a logarithm (medium)

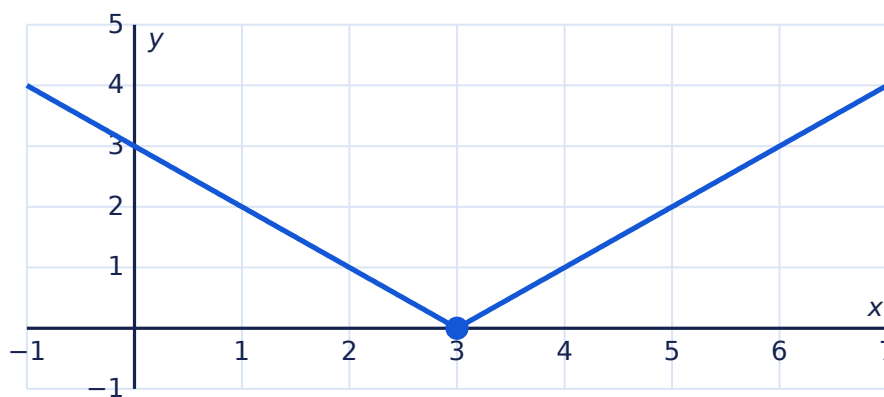
Let $f(x) = \frac{(\ln x)}{x^2}$. Find $f'(x)$ and evaluate $f'(e)$.

Problem 14 Differentiable piecewise function (medium)

Let $f(x) = x^2$ for $x \leq 3$ and $f(x) = ax + b$ for $x > 3$. Find a and b so that f is differentiable at $x = 3$.

Problem 15 Checking differentiability at a corner (hard)

Let $f(x) = |x - 3|$. Find the slope of the graph just to the left and just to the right of $x = 3$, and decide whether $f'(3)$ exists.



Graph of f .

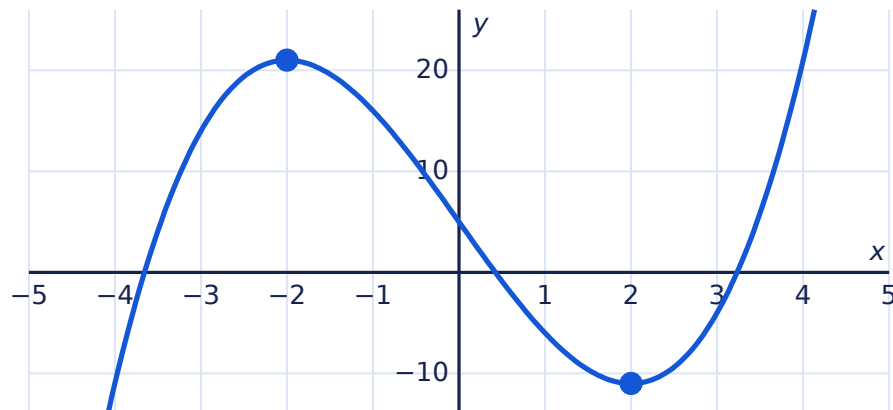
Problem 16 Estimating a derivative from a table (hard)

The depth $D(t)$ of a river at a gauge, in feet, is recorded. Use the points nearest $t = 4$ to estimate $D'(4)$, with units.

t (hours)	2	4	6	9
$D(t)$	18	29	44	62

Problem 17 Horizontal tangent lines (hard)

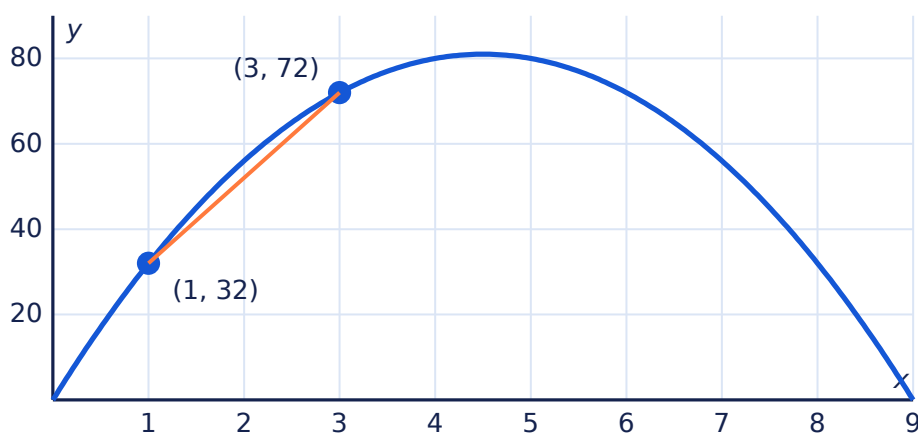
Find every point on the graph of $f(x) = x^3 - 12x + 5$ where the tangent line is horizontal.



Graph of f .

Problem 18 Drone height (hard)

A drone's height in feet is $h(t) = -4t^2 + 36t$ after t seconds. Find its average velocity on $[1, 3]$, its velocity at $t = 2$, and the time when its velocity is zero.



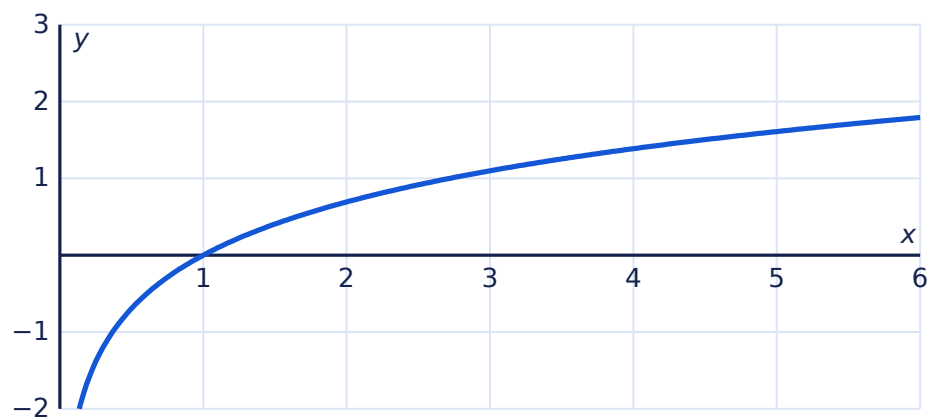
Height of the drone.

Problem 19 Concentration of a medicine (hard)

The concentration of a drug in the blood is $C(t) = \frac{8t}{(t^2 + 4)}$ mg/L, t hours after injection. Find $C'(t)$, evaluate $C'(1)$, and find when the concentration stops rising.

Problem 20 Tangent line through the origin (hard)

A line through the origin is tangent to the graph of $f(x) = \ln x$. Find the point of tangency and the equation of the line.



Graph of f .