

Modeling Change with Separable Equations

Grade 12 – AP Calculus AB math · Lesson · Standards: AP Calculus AB Unit 7

Quick answer: A differential equation relates a function to its derivative. Slope fields show the direction of solutions at each point, separation of variables solves equations of the form $dy/dx = f(x)g(y)$, and exponential models solve $dy/dt = ky$, giving growth and decay with $y = y_0 e^{(kt)}$.

Standards: AP Calculus AB Unit 7 · **Level:** Grade 12 AP Calculus AB

What is a differential equation?

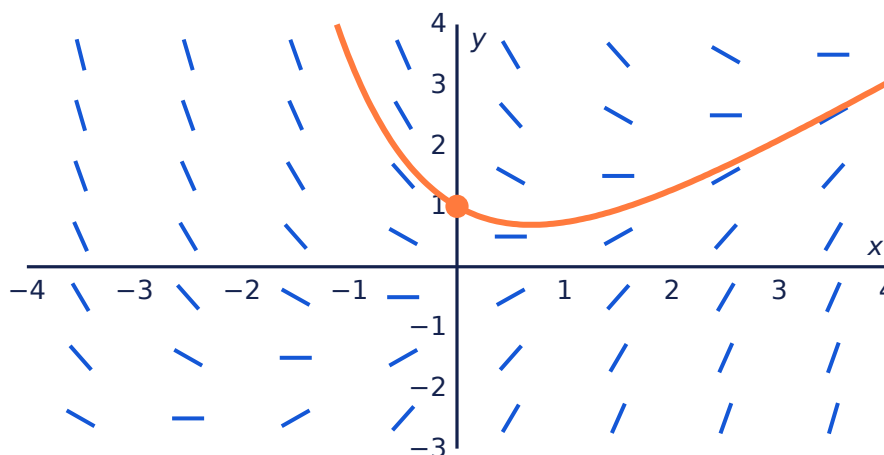
Differential equation

A differential equation relates a function to its own derivative, such as $dy/dx = 2xy$. A **solution** is a function that makes the equation true. A **particular solution** also meets an initial condition like $y(0) = 3$.

To verify a solution, differentiate it and substitute both the function and its derivative into the equation. For $dy/dx = 3y$, the function $y = 5e^{3x}$ works because $y' = 15e^{3x} = 3(5e^{3x})$.

How do slope fields show solutions?

A slope field draws a short segment with slope dy/dx at each grid point. A solution curve follows the segments like a boat following a current. Where the formula gives 0, the segments are horizontal.



For $dy/dx = x - y$, segments are horizontal along $y = x$. The orange curve is the solution through $(0, 1)$.

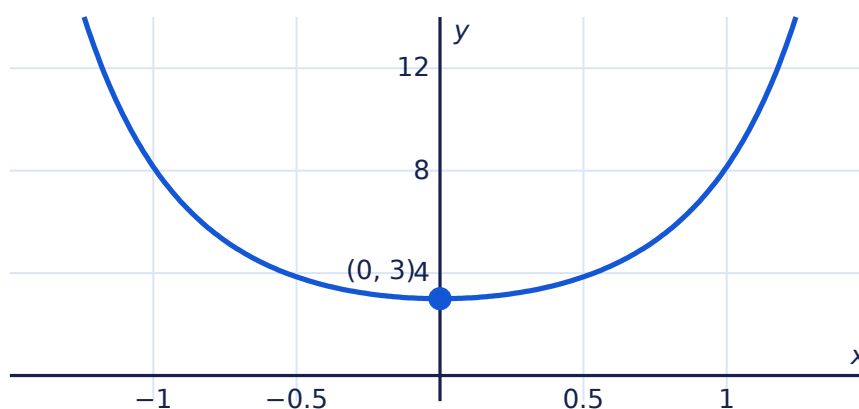
How does separation of variables work?

Separation of variables

1. Move every y to one side with dy and every x to the other with dx .
2. Integrate both sides and include one constant C .
3. Use the initial condition to find C , then solve for y if possible.

Worked example

Solve $\frac{dy}{dx} = 2xy$ with $y(0) = 3$. Separate: $dy/y = 2x \, dx$. Integrate: $\ln|y| = x^2 + C$. So $y = Ae^{x^2}$. Since $y(0) = 3$, $A = 3$ and the solution is $y = 3e^{x^2}$.

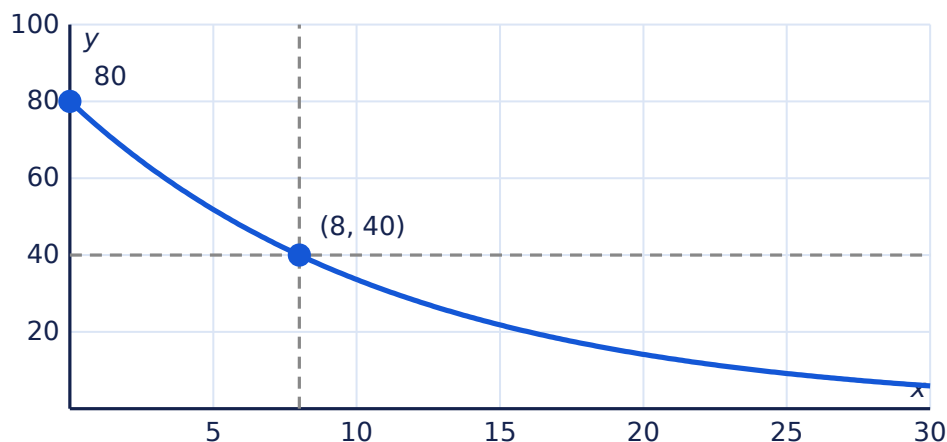


The particular solution passes through $(0, 3)$.

What is exponential growth and decay?

Exponential model

If the rate of change is proportional to the amount, $\frac{dy}{dt} = ky$, then $y = y_0 e^{kt}$. Growth has $k > 0$ and decay has $k < 0$. A half-life T gives $k = -(\ln 2)/T$.



A sample of 80 grams with half-life 8 days: after 8 days, 40 grams remain.

Common mistake

Forgetting the constant C at the moment of integration loses the initial condition. Add C immediately, before solving for y .

Mathbot's tip

Verify every solution by differentiating it. It takes ten seconds and catches sign errors.

Key points to remember

- A solution makes the differential equation true; a particular solution also matches an initial condition.
- A slope field draws the value of dy/dx as a short segment at each grid point.
- Separation of variables: move y to one side and x to the other, then integrate both.
- If $dy/dt = ky$, then $y = y_0 e^{(kt)}$.
- Always add the constant C when integrating, before using the initial condition.

Modeling Change with Separable Equations: frequently asked questions

How do I check that a function solves a differential equation?

Differentiate the function, then substitute the function and its derivative into the equation. If both sides become identical, the function is a solution.

What does a slope field tell me?

Each segment shows the slope of a solution passing through that point. You can sketch a solution by starting at the initial point and following the segments.

When can I use separation of variables?

Use it when the derivative can be written as a product of a function of x alone and a function of y alone. Then integrate each side separately.

How do I find the constant of proportionality from a half-life?

Set the amount equal to half of the start at the half-life time: $e^{kT} = 1/2$, so $k = -(\ln 2)/T$. A negative k indicates decay.

What is an equilibrium solution?

It is a constant solution where the derivative is zero, so the slope field has horizontal segments all along that line. For $dy/dx = y(4 - y)$ the equilibria are $y = 0$ and $y = 4$.