

Modeling Change with Separable Equations

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 7

Worked solutions and scoring guides follow. Award separation and integration points independently, and require the constant of integration. Answers without an initial-condition step cannot earn full credit on particular-solution questions.

Standards: AP Calculus AB Unit 7 · **Level:** Grade 12 AP Calculus AB

Question 1 Solution with an initial condition (2 points)

Write $\frac{dy}{dx} = 3(y - 2)$. Separation gives $y - 2 = Ae^{3x}$. With $y(0) = 7$, $A = 5$. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for a correct separated form.

Question 2 Matching a slope field (2 points)

The segments are horizontal along $y = x$, so the slope involves $x - y$. At $(0, 2)$ the segment slopes steeply downward: $x - y = -2$. This rules out (C), whose slope there would be $+2$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for checking a point.

Question 3 Separable equation (3 points)

(a) Separate: $y \, dy = 6x^2 \, dx$. Integrate: $y^2/2 = 2x^3 + C$.

(b) At $(1, 3)$: $9/2 = 2 + C$, so $C = 5/2$. Then $y^2 = 4x^3 + 5$. Since $y(1) > 0$, $y = \sqrt{(4x^3 + 5)}$.

Scoring: Part (a): 1 point for separating, 1 point for integrating both sides with a constant. Part (b): 1 point.

Question 4 Radioactive decay (3 points)

(a) $m(t) = 250e^{-0.04t}$.

(b) Solve $e^{-0.04t} = 1/2$: $t = \ln 2/0.04 \approx 17.3$ years.

(c) Solve $40 = 250e^{-0.04t}$: $t = \ln(6.25)/0.04 \approx 45.8$ years.

Scoring: 1 point per part.

Question 5 Verifying a solution (3 points)

(a) $y' = 2e^{2x} + 1$. Then $y' - 2y = 2e^{2x} + 1 - 2e^{2x} - 2x = 1 - 2x$. **The equation holds.**

(b) $y(0) = e^0 + 0 = \mathbf{1}$.

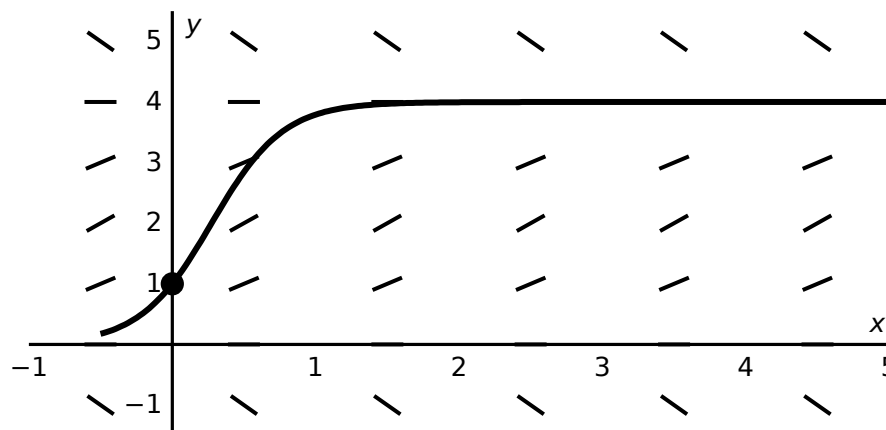
Scoring: Part (a): 1 point for the derivative, 1 point for the substitution. Part (b): 1 point.

Question 6 Slope field reasoning (3 points)

(a) The slope is zero for $y = 0$ and $y = 4$.

(b) The solution starts at 1 with positive slope and cannot cross the equilibrium $y = 4$, so the limit is **4**.

(c) $\frac{d^2y}{dx^2} = (4 - 2y)\frac{dy}{dx}$. For $0 < y < 2$, both factors are positive, so the solution is **concave up**.



The solution through $(0, 1)$.

Scoring: 1 point per part.

Question 7 Cooling model (2 points)

(a) Solving gives $T = 20 + 60e^{-0.1t}$. So $T(5) = 20 + 60e^{-0.5} \approx \mathbf{56.4^\circ\text{F}}$.

(b) As $t \rightarrow \infty$, $e^{-0.1t} \rightarrow 0$, so T approaches **20°F** .

Scoring: 1 point per part.

Question 8 Tangent line approximation (2 points)

(a) The slope is $x^2 - y = 1 - 2 = -1$ at $(1, 2)$. The tangent line is **$y = 2 - (x - 1)$** .

(b) At $x = 1.1$: $2 - 0.1 = \mathbf{1.9}$.

Scoring: 1 point per part.