

Modeling Change with Separable Equations

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 7

Name: _____ Date: _____ Class: _____

This 45-minute test follows the AP Calculus AB style: two multiple-choice items and free-response questions requiring a full solution. Show every step. The total is 20 points, and no calculator is needed except for numerical answers.

Standards: AP Calculus AB Unit 7 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

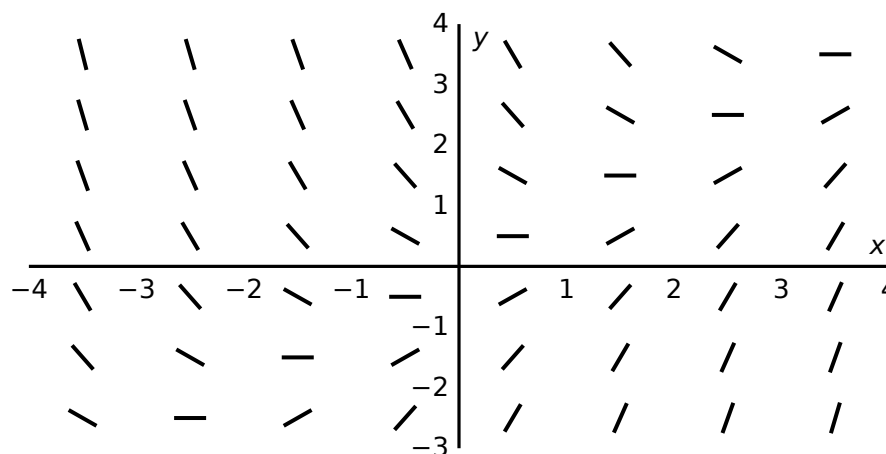
Question 1 Solution with an initial condition (2 points)

Which function is the solution of $\frac{dy}{dx} = 3y - 6$ with $y(0) = 7$?

- A. $2 + 5e^{3x}$
- B. $2 + 7e^{3x}$
- C. $7e^{3x}$
- D. $7 + e^{3x}$

Question 2 Matching a slope field (2 points)

Which differential equation matches the slope field shown?



Slope field.

- A. $\frac{dy}{dx} = x + y$

- B. $\frac{dy}{dx} = x - y$
- C. $\frac{dy}{dx} = y - x$
- D. $\frac{dy}{dx} = xy$

Question 3 Separable equation (3 points)

Consider $\frac{dy}{dx} = 6x^2/y$ with $y(1) = 3$.

- a. Separate the variables and integrate both sides. (2 points)
- b. Find the particular solution. (1 point)

Question 4 Radioactive decay (3 points)

A radioactive sample of mass m (grams) satisfies $dm/dt = -0.04m$ with $m(0) = 250$, t in years.

- a. Write $m(t)$. (1 point)
- b. Find the half-life. (1 point)
- c. When will 40 grams remain? (1 point)

Question 5 Verifying a solution (3 points)

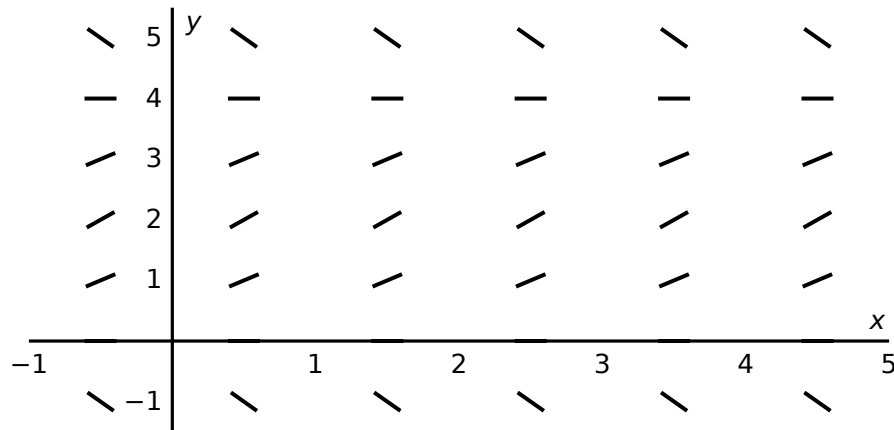
Let $y = e^{2x} + x$.

- a. Show that y satisfies $y' - 2y = 1 - 2x$. (2 points)
- b. Show that y satisfies the initial condition $y(0) = 1$. (1 point)

Question 6 Slope field reasoning (3 points)

The slope field for $\frac{dy}{dx} = y(4 - y)$ is shown.

- a. Name the equilibrium solutions. (1 point)
- b. For the solution with $y(0) = 1$, find the limit of y as $x \rightarrow \infty$. (1 point)
- c. Is the solution concave up or concave down where $0 < y < 2$? Justify. (1 point)



Slope field.

Question 7 Cooling model (2 points)

A drink at 80°F is placed in a 20°F refrigerator. Its temperature T satisfies $dT/dt = -0.1(T - 20)$, t in minutes.

- Find $T(5)$. (1 point)
- What does T approach as $t \rightarrow \infty$? (1 point)

Question 8 Tangent line approximation (2 points)

The function $y = f(x)$ satisfies $dy/dx = x^2 - y$ with $f(1) = 2$.

- Write the equation of the tangent line at $x = 1$. (1 point)
- Use it to approximate $f(1.1)$. (1 point)