

Modeling Change with Separable Equations

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 7

Name: _____ Date: _____ Class: _____

These 20 problems cover AP Calculus AB Unit 7: verifying solutions, slope fields, separation of variables, exponential growth and decay, and particular solutions from initial conditions. Early problems verify or evaluate; later ones solve full models.

The last six problems are applied or multi-step, covering mixing, draining, cooling and the choice of sign when solving for a root.

Standards: AP Calculus AB Unit 7 · **Level:** Grade 12 AP Calculus AB

Problem 1 Verifying an exponential solution (easy)

Show that $y = 3e^{2x}$ is a solution of $\frac{dy}{dx} = 2y$.

Problem 2 General and particular solutions (easy)

Show that $y = x^2 + C$ solves $\frac{dy}{dx} = 2x$, and find the particular solution with $y(1) = 5$.

Problem 3 Verifying a second-order solution (medium)

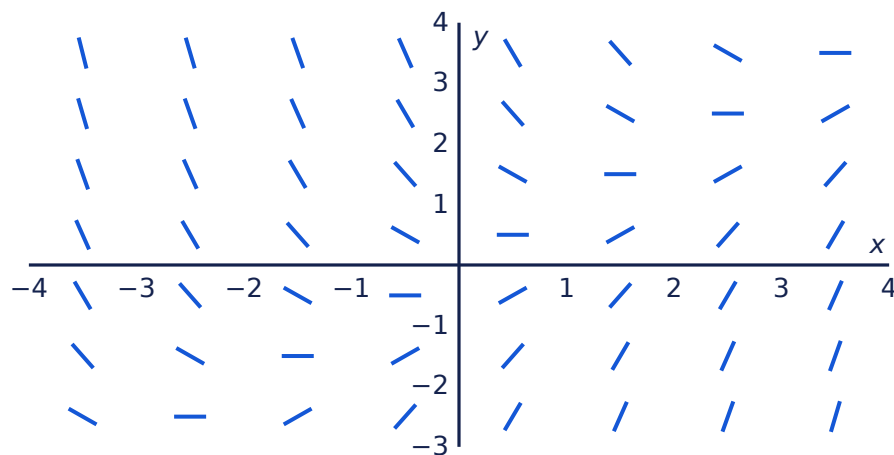
Show that $y = 5 \sin 2x + x$ satisfies $\frac{d^2y}{dx^2} + 4y = 4x$.

Problem 4 Computing slopes from an equation (easy)

For $\frac{dy}{dx} = x + y$, find the slope at $(1, 2)$, at $(-2, 1)$ and at $(0, 0)$.

Problem 5 Reading a slope field (medium)

The slope field for $\frac{dy}{dx} = x - y$ is shown. Along which line are all the slopes zero? Is the solution through $(0, 1)$ increasing or decreasing at $x = 0$?



Slope field.

Problem 6 Separable equation with a power (medium)

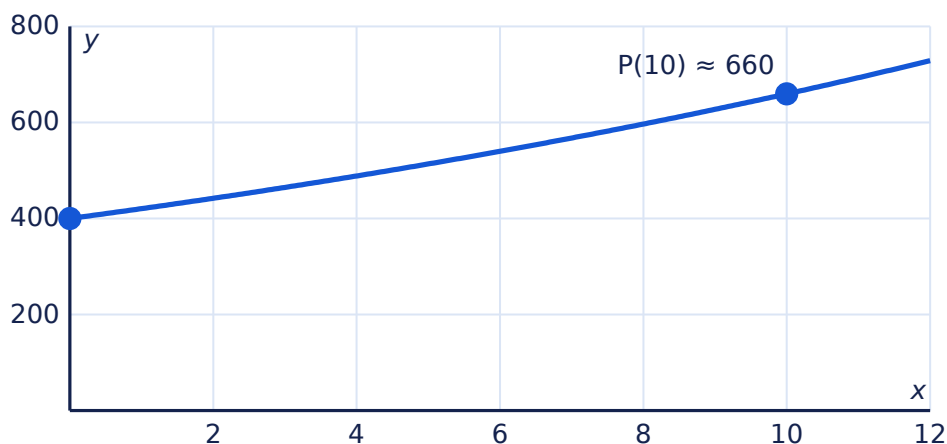
Find the particular solution of $\frac{dy}{dx} = 4x^3y$ with $y(0) = 5$.

Problem 7 Separable equation with a quotient (medium)

Solve $\frac{dy}{dx} = \frac{x}{y}$ with $y(2) = 3$.

Problem 8 Population growth model (medium)

A bacteria culture satisfies $\frac{dP}{dt} = 0.05P$ with $P(0) = 400$, where t is in hours. Find $P(t)$ and $P(10)$.



Graph of the culture size.

Problem 9 Half-life (medium)

A 90-milligram sample of a medicine decays at a rate proportional to the amount present, with a half-life of 8 days. How many milligrams remain after 20 days?

Problem 10 Equation with an exponential (medium)

Find the particular solution of $\frac{dy}{dx} = e^{x-y}$ with $y(0) = \ln 3$.

Problem 11 Equation with a trigonometric factor (medium)

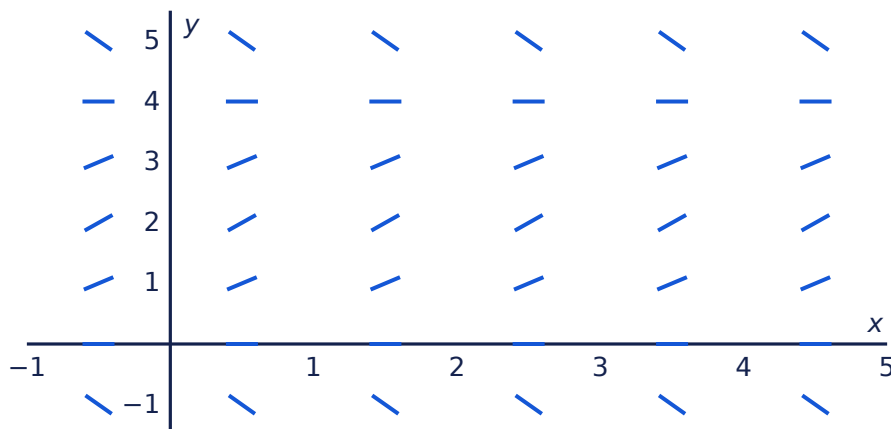
Find the solution of $\frac{dy}{dx} = y \sin x$ that satisfies $y(0) = 3$.

Problem 12 Newton's law of cooling (hard)

A cup of coffee at 190°F is placed in a 70°F room. Its temperature T obeys $dT/dt = -0.1(T - 70)$ with t in minutes. Find $T(t)$ and the temperature after 10 minutes.

Problem 13 Equilibrium and long-term behavior (hard)

The slope field for $\frac{dy}{dx} = y(4 - y)$ is shown. Name the equilibrium solutions, and describe the limit of the solution with $y(0) = 1$ as $x \rightarrow \infty$. Where is that solution concave up?



Slope field.

Problem 14 Separable equation with a cube root (hard)

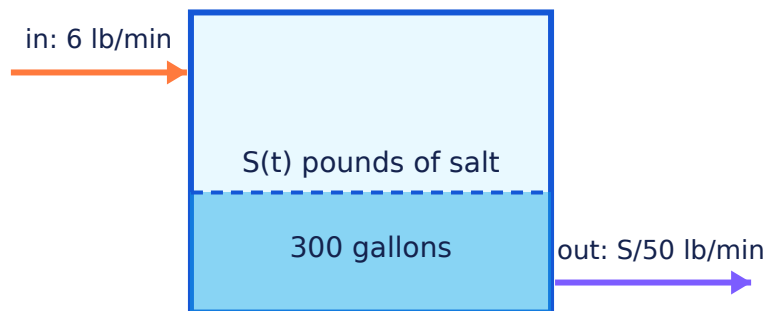
Find the particular solution of $\frac{dy}{dx} = \frac{(x+1)^2}{y^2}$ with $y(0) = 2$.

Problem 15 Fish population (hard)

A lake has 2,400 fish, and the population grows at a rate proportional to its size. After 3 years there are 3,100. Find the growth constant k and predict the population after 10 years.

Problem 16 Salt in a tank (hard)

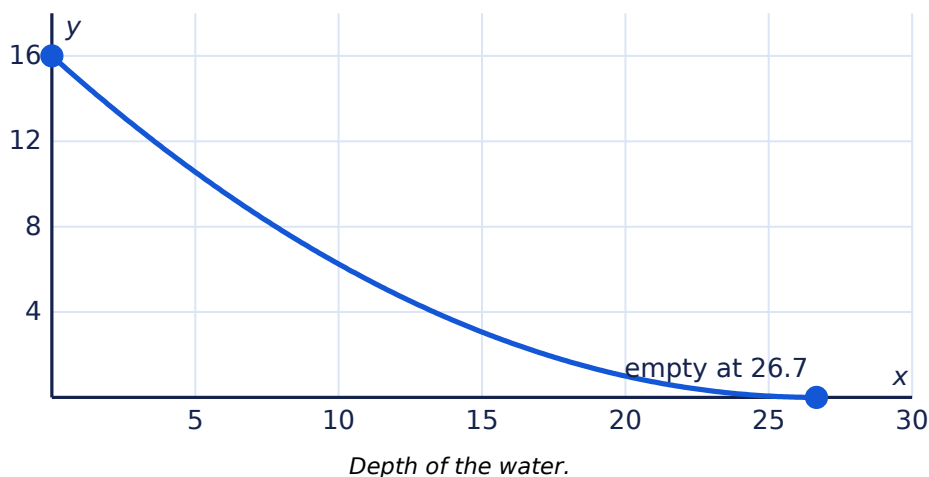
A 300-gallon tank starts with pure water. Salt solution enters, adding 6 pounds of salt per minute, and the well-mixed solution leaves so that the salt amount S (pounds) obeys $dS/dt = 6 - S/50$. Find $S(t)$, the amount after 50 minutes and the long-term amount.



The mixing tank.

Problem 17 Draining tank (hard)

Water drains from a tank so that its depth h (feet) satisfies $dh/dt = -0.3\sqrt{h}$ with $h(0) = 16$, t in minutes. Find $h(t)$ and the time when the tank is empty.

**Problem 18 Choosing the sign of a root** (hard)

Solve $dy/dx = (x+2)/y$ with $y(0) = -1$.

Problem 19 Equilibrium and exponential behavior (hard)

Solve $dy/dx = x(y-3)$ with $y(0) = 5$. Identify the equilibrium solution and find the value of $y(2)$.

Problem 20 Time to cool (hard)

Tea at 96°F sits in a 22°F porch. Its temperature satisfies $dT/dt = -0.08(T - 22)$. When does the tea reach 50°F? Will it ever reach 22°F?