

Integrals, Riemann Sums and Accumulation

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 6

Worked solutions and scoring guides follow. Credit a correct antiderivative or setup even when later arithmetic slips, but require units for contextual answers and a stated reason where the question asks for a justification.

Standards: AP Calculus AB Unit 6 · **Level:** Grade 12 AP Calculus AB

Question 1 Definite integral (2 points)

An antiderivative is $x^3 - x^2$. At 2: $8 - 4 = 4$; at 0: 0. The value is 4. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for a correct antiderivative.

Question 2 Accumulation derivative (2 points)

By FTC Part 1, $H'(x) = \sqrt{x^4 + 9}$. At $x = 2$: $\sqrt{25} = 5$. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for using FTC Part 1.

Question 3 Riemann sums from a table (3 points)

(a) $8(3) + 10(2) + 7(4) = 24 + 20 + 28 = \mathbf{72}$.

(b) $10(3) + 7(2) + 2(4) = 30 + 14 + 8 = \mathbf{52}$.

(c) $(8 + 10)/2 \cdot 3 + (10 + 7)/2 \cdot 2 + (7 + 2)/2 \cdot 4 = 27 + 17 + 18 = \mathbf{62 \text{ gallons}}$.

Scoring: 1 point per part; units required in (c).

Question 4 Substitution (3 points)

(a) Let $u = x^3 + 2$, so $du = 3x^2 dx$. Then $(1/3) \int u^4 du = u^5/15 + C = \mathbf{(x^3 + 2)^5/15 + C}$.

(b) $[(x^3 + 2)^5/15]$ from 0 to 1 = $(3^5 - 2^5)/15 = (243 - 32)/15 = \mathbf{211/15}$.

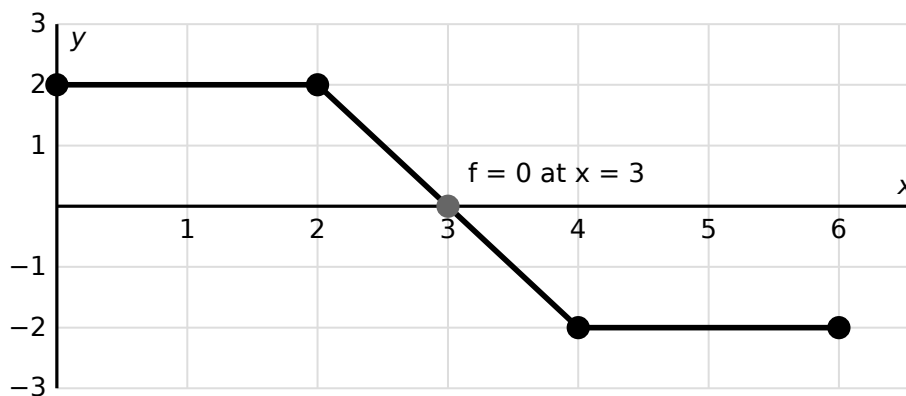
Scoring: Part (a): 1 point for the substitution, 1 point for the antiderivative. Part (b): 1 point.

Question 5 Accumulation function from a graph (3 points)

(a) $g(2) = 2 \cdot 2 = 4$. From 2 to 4 the net signed area is $(2 + (-2))/2 \cdot 2 = 0$, so $g(4) = 4$.

(b) From 4 to 6 the area is $(-2)(2) = -4$, so $g(6) = 0$.

(c) $g' = f$ changes from positive to negative at $x = 3$ (the line from $(2, 2)$ to $(4, -2)$ crosses zero there). $g(3) = 4 + (2 + 0)/2 \cdot 1 = 5$. Comparing with $g(0) = 0$ and $g(6) = 0$, the maximum is **5**.



The graph crosses zero at $x = 3$.

Scoring: 1 point per part.

Question 6 Long division and completing the square (3 points)

(a) Divide: $x^2 - 3x + 5 = (x - 1)(x - 2) + 3$, so the integrand is $x - 2 + 3/(x - 1)$. The antiderivative is $x^2/2 - 2x + 3 \ln|x - 1| + C$.

(b) $x^2 + 4x + 5 = (x + 2)^2 + 1$, so the antiderivative is $\arctan(x + 2) + C$.

Scoring: Part (a): 1 point for the division, 1 point for the antiderivative. Part (b): 1 point.

Question 7 Properties of integrals (2 points)

(a) $\int_4^6 = -2 - 7 = -9$.

(b) $\int_1^4 (2f(x) + 3) dx = 2(7) + 3(3) = 23$. Reversing the limits gives **-23**.

Scoring: 1 point per part.

Question 8 Net change in a tank (2 points)

(a) $40 + \int_0^6 (12 - 2t) dt = 40 + (72 - 36) = 76$ gallons.

(b) The amount increases while $R > 0$ and decreases when $R < 0$. R changes from positive to

negative at $t = 6$, so the greatest amount occurs at **$t = 6$** .

Scoring: 1 point per part.