

Integrals, Riemann Sums and Accumulation

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 6

Name: _____ Date: _____ Class: _____

This 45-minute test follows the AP Calculus AB style: two multiple-choice items and free-response questions with units and justification. Show every step. The total is 20 points, and no calculator is needed.

Standards: AP Calculus AB Unit 6 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

Question 1 Definite integral (2 points)

What is the value of $\int_0^2 (3x^2 - 2x) dx$?

- A. 4
- B. 6
- C. 2
- D. 8

Question 2 Accumulation derivative (2 points)

Let $H(x) = \int_0^x \sqrt{t^4 + 9} dt$. What is $H'(2)$?

- A. 5
- B. 25
- C. 3
- D. 1/5

Question 3 Riemann sums from a table (3 points)

The rate $r(t)$ at which a machine fills a tank, in gallons per minute, is given by the table.

t (min)	0	3	5	9
$r(t)$	8	10	7	2

- a. Find the left Riemann sum for $\int_0^9 r(t) dt$ using the three subintervals. (1 point)
- b. Find the right Riemann sum using the same subintervals. (1 point)

c. Find the trapezoidal sum and give the units. (1 point)

Question 4 Substitution (3 points)

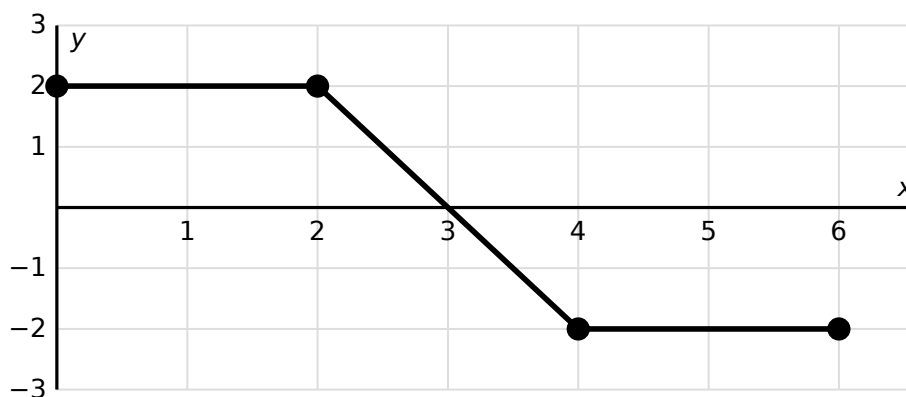
Let $f(x) = x^2(x^3 + 2)^4$.

- Find $\int f(x) dx$. (2 points)
- Evaluate $\int_0^1 f(x) dx$. (1 point)

Question 5 Accumulation function from a graph (3 points)

The graph of f consists of line segments. Let $g(x) = \int_0^x f(t) dt$.

- Find $g(2)$ and $g(4)$. (1 point)
- Find $g(6)$. (1 point)
- Find the absolute maximum value of g on $[0, 6]$ and justify. (1 point)



Graph of f .

Question 6 Long division and completing the square (3 points)

Find each antiderivative.

- $\int \frac{(x^2 - 3x + 5)}{(x - 1)} dx$ (2 points)
- $\int \frac{1}{(x^2 + 4x + 5)} dx$ (1 point)

Question 7 Properties of integrals (2 points)

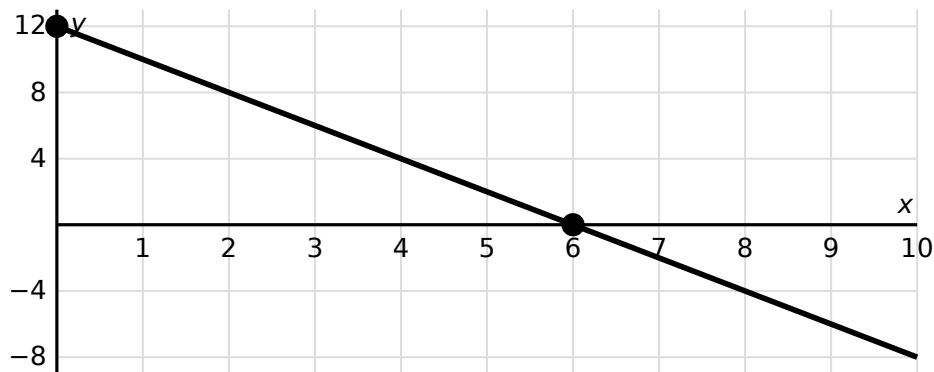
For a function f , $\int_1^4 f(x) dx = 7$ and $\int_1^6 f(x) dx = -2$.

- Find $\int_4^6 f(x) dx$. (1 point)
- Find $\int_4^1 (2f(x) + 3) dx$. (1 point)

Question 8 Net change in a tank (2 points)

Water enters a tank at rate $R(t) = 12 - 2t$ gallons per minute for $0 \leq t \leq 10$ (a negative value means water is leaving). The tank holds 40 gallons at $t = 0$.

- How many gallons are in the tank at $t = 6$? (1 point)
- At what time on $[0, 10]$ is the amount of water greatest? Justify. (1 point)



Graph of $R(t)$.