

Integrals, Riemann Sums and Accumulation

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 6

Name: _____ Date: _____ Class: _____

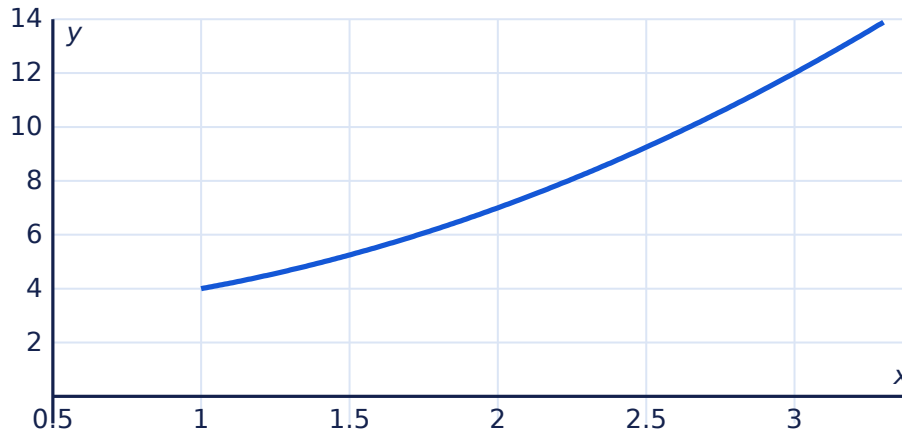
These 20 problems cover AP Calculus AB Unit 6: Riemann sums, definite integrals, the Fundamental Theorem of Calculus, antiderivatives, u-substitution and accumulation functions. Early problems build the basic techniques; later ones mix several ideas.

The last six problems are multi-step: a midpoint estimate of distance, displacement versus distance, and an accumulation function analyzed with the second derivative.

Standards: AP Calculus AB Unit 6 · **Level:** Grade 12 AP Calculus AB

Problem 1 Left and right Riemann sums (easy)

Use four subintervals of equal width to find the left and right Riemann sums for $f(x) = x^2 + 3$ on $[1, 3]$.



Graph of f .

Problem 2 Trapezoidal sum from a table (medium)

Water flows into a pond at rate $r(t)$ gallons per minute. Use a trapezoidal sum with four subintervals to estimate the water gained from $t = 0$ to 8 minutes.

t (min)	0	2	4	6	8
$r(t)$	5	8	9	7	4

Problem 3 Antiderivative of a polynomial (easy)

Find $\int (6x^2 - 4x + 7) dx$.

Problem 4 Antiderivative with ln and exponential (easy)

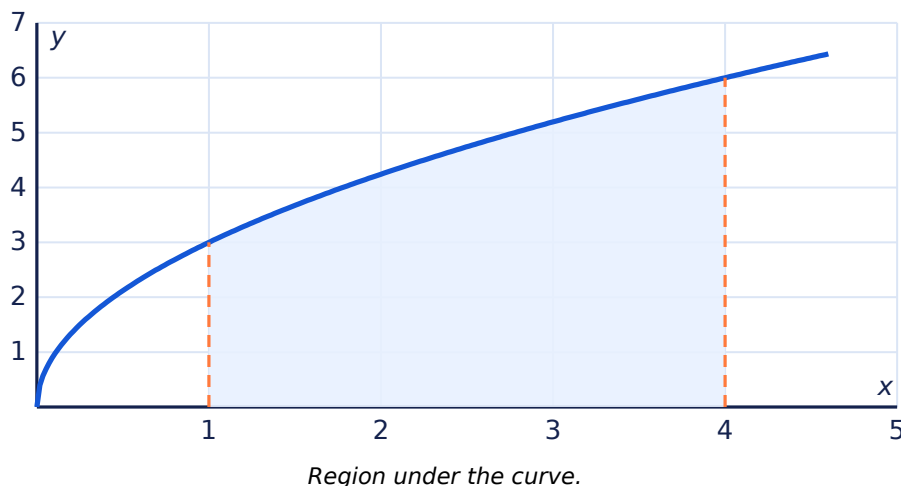
Find $\int (\frac{3}{x} + e^x - \sin x) dx$.

Problem 5 Meaning of a definite integral (easy)

A food truck sells burritos at a rate of $s(t)$ burritos per hour, t hours after opening, and $\int_2^6 s(t) dt = 148$. Explain what the number 148 means.

Problem 6 Evaluating with the Fundamental Theorem (medium)

Evaluate $\int_1^4 3\sqrt{x} dx$.

**Problem 7 Trigonometric definite integral** (medium)

Evaluate $\int_0^\pi (2 \cos x + 1) dx$.

Problem 8 Properties of definite integrals (medium)

Suppose $\int_0^5 f(x) dx = 12$ and $\int_0^2 f(x) dx = -3$. Find

- a. $\int_2^5 f(x) dx$
- b. $\int_5^0 f(x) dx$
- c. $\int_0^5 (4f(x) - 2) dx$

Problem 9 Substitution with a power (medium)

Find $\int 2x(x^2 + 5)^4 dx$.

Problem 10 Substitution with an exponential (medium)

Find $\int x^2 e^{x^3} dx$.

Problem 11 Definite integral with substitution (hard)

Evaluate $\int_0^{\sqrt{7}} x\sqrt{x^2 + 9} dx$.

Problem 12 Derivative of an accumulation function (medium)

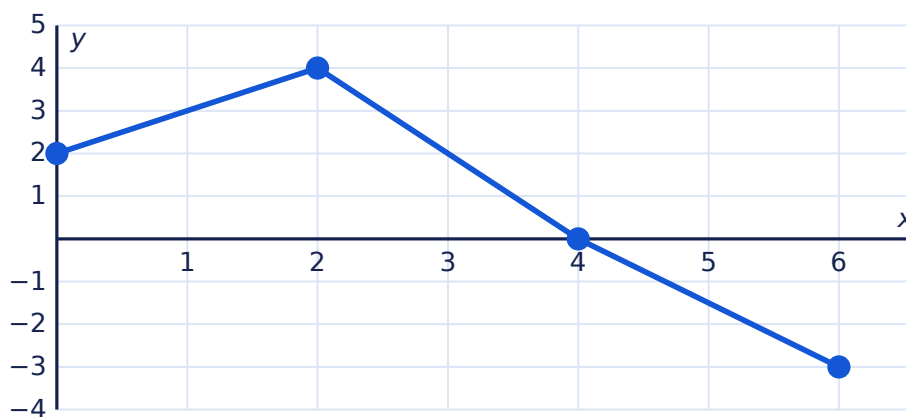
Let $G(x) = \int_1^x \sqrt{t^3 + 8} dt$. Find $G(1)$ and $G'(2)$.

Problem 13 Chain rule with an integral (hard)

Let $H(x) = \int_1^{x^2} (3t + 1) dt$. Find $H'(x)$ and $H'(2)$.

Problem 14 Accumulation from a graph (hard)

The graph of f is made of line segments. Let $g(x) = \int_0^x f(t) dt$. Find $g(2)$, $g(4)$, $g(6)$, and the x -value where g is largest.



Graph of f .

Problem 15 Integrating after long division (hard)

Find $\int \frac{(x^2 + 5x + 7)}{(x + 1)} dx$.

Problem 16 Completing the square (hard)

Find $\int \frac{1}{x^2 + 6x + 13} dx$.

Problem 17 Logarithm substitution (hard)

Evaluate $\int_1^{e^2} \frac{(\ln x)}{x} dx$.

Problem 18 Distance from a speed table (hard)

A cyclist's speed $v(t)$, in miles per hour, is recorded every 2 hours. Use a midpoint sum with three subintervals of equal width to estimate the distance traveled from $t = 0$ to 12 hours.

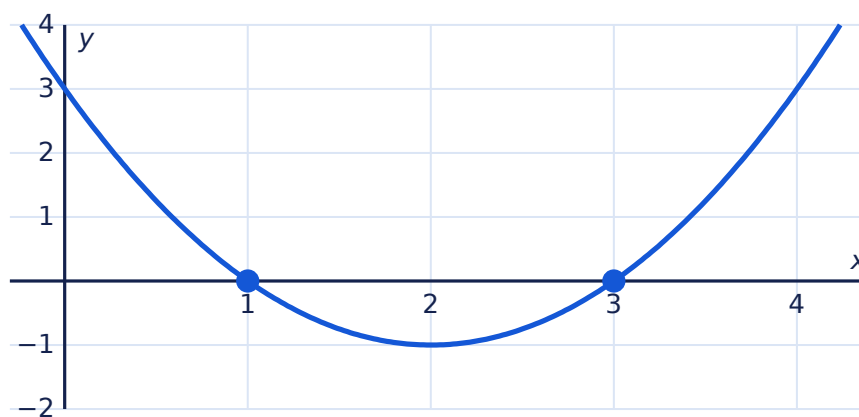
t (h)	0	2	4	6	8	10	12
$v(t)$	0	18	32	40	44	45	43

Problem 19 Displacement and distance (hard)

A particle on a line has velocity $v(t) = 3t^2 - 12t + 9$ for $0 \leq t \leq 4$. Find its displacement and the total distance traveled.

Problem 20 Analyzing an accumulation function (hard)

Let $f(x) = (x - 1)(x - 3)$ and $g(x) = \int_0^x f(t) dt$. Find the critical points of g , classify them with the second derivative, and find g at each.



Graph of f .