

Limits and Continuity: Approaching a Value

Grade 12 – AP Calculus AB math · Lesson · Standards: AP Calculus AB Unit 1

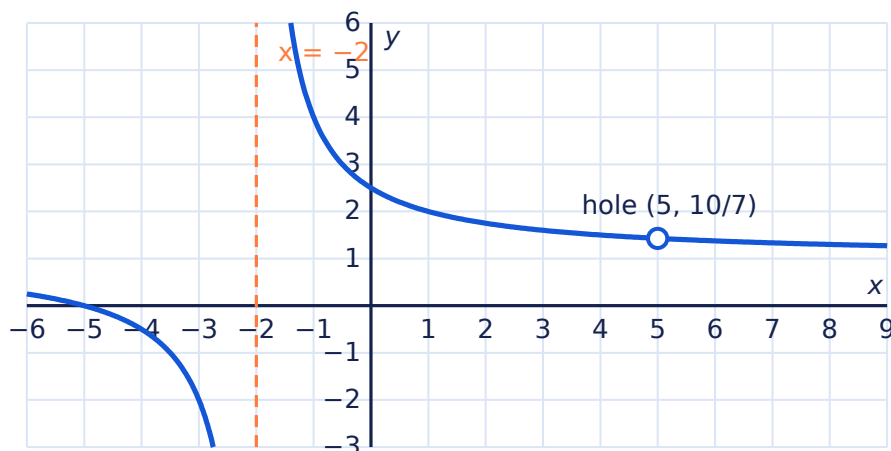
Quick answer: A limit describes the value a function approaches as its input gets close to a number, whether or not the function reaches that value. In AP Calculus AB, limits are found from graphs, tables and algebra, and they define continuity, asymptotes and, later, the derivative.

Standards: AP Calculus AB Unit 1 · **Level:** Grade 12 AP Calculus AB

What does a limit tell you about a function?

Limit

The limit of $f(x)$ as x approaches a is the single value that $f(x)$ gets arbitrarily close to when x is close to a but not equal to a . If the left-hand and right-hand limits disagree, the limit does not exist.



Near $x = 5$ the graph has a hole at height $10/7$, and near $x = -2$ it climbs without bound.

How do you evaluate a limit algebraically?

Method for a 0/0 form

1. Substitute the number. If you get a real value, you are finished.
2. If you get $0/0$, factor the numerator and denominator, or multiply by a conjugate.
3. Cancel the common factor, then substitute again.

Worked example

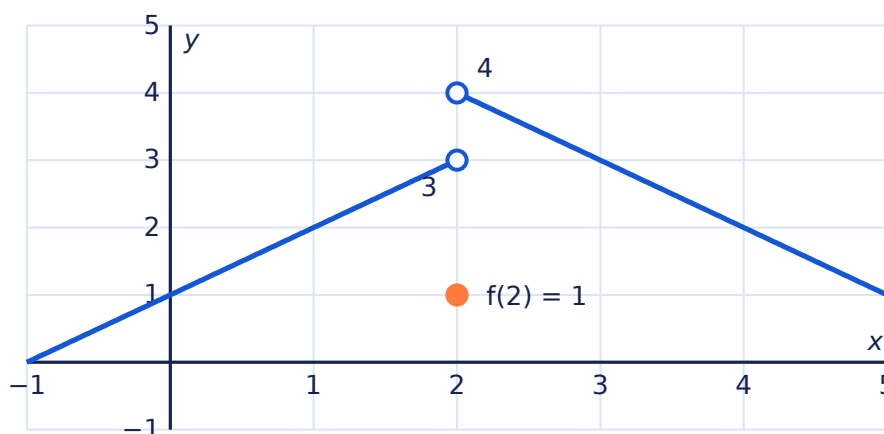
$\lim_{x \rightarrow 5} \frac{(x^2 - 25)}{(x^2 - 3x - 10)}$. Substituting gives 0/0. Factor: $\frac{((x - 5)(x + 5))}{((x - 5)(x + 2))} = \frac{(x + 5)}{(x + 2)}$. Now substitute 5 to get **10/7**, which matches the hole in the figure.

When is a function continuous?

Three-part test at $x = a$

$f(a)$ exists, $\lim_{x \rightarrow a} f(x)$ exists, and the two are equal. Failing the second or third part gives one of three discontinuities.

Type	What happens	Example
Removable	Limit exists but differs from $f(a)$ or $f(a)$ is missing	Hole at $x = 5$ above
Jump	Left and right limits are different finite numbers	Figure below, at $x = 2$
Infinite	Function grows without bound, giving a vertical asymptote	Line $x = -2$ above



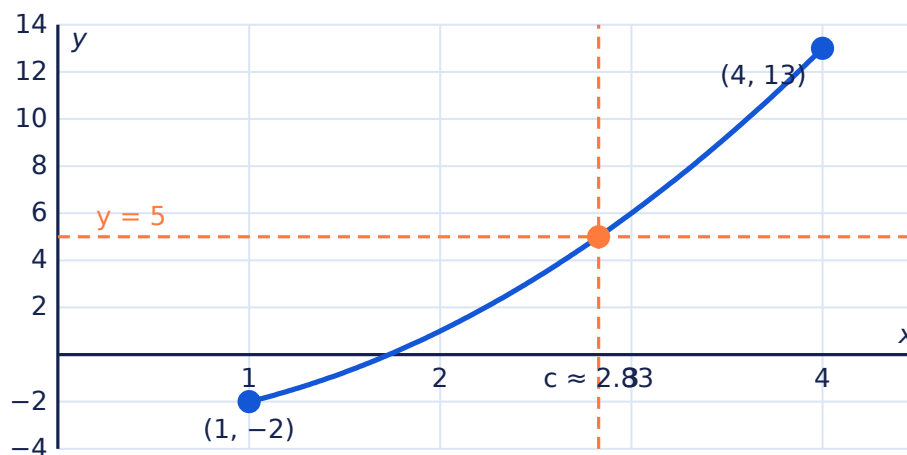
A jump at $x = 2$: the one-sided limits are 3 and 4, so the limit does not exist.

What do limits at infinity and the Intermediate Value Theorem say?

For a rational function, compare the highest powers. The limit as $x \rightarrow \infty$ of $\frac{6x^2}{(2x^2 + 5)}$ is $6/2 = 3$, so the line $y = 3$ is a horizontal asymptote. The Squeeze Theorem says that if $f(x)$ is trapped between two functions with the same limit L , the trapped function also has limit L .

Intermediate Value Theorem

If $f(x)$ is continuous on $[a, b]$ and N lies between $f(a)$ and $f(b)$, then $f(c) = N$ for at least one c in (a, b) .



With $f(x) = x^2 - 3$ on $[1, 4]$, the value 5 lies between -2 and 13 , so the graph must cross $y = 5$.

Common mistake

Continuity needs a continuous function on the *closed* interval. A gap or asymptote inside the interval voids the guarantee.

Mathbot's tip

A limit asks what the function is heading toward, not what it equals. Cover up the point itself and look at both sides.

Key points to remember

- A two-sided limit exists only when the left-hand and right-hand limits are equal.
- A $0/0$ form means algebra is needed: factor, conjugate, then substitute.
- Continuity at a point needs a defined value, an existing limit, and equality of the two.
- A rational function has a horizontal asymptote determined by its leading terms.
- The Intermediate Value Theorem needs continuity on a closed interval.

Limits and Continuity: Approaching a Value: frequently asked questions

What does it mean when a limit does not exist?

It means the function does not settle on one finite number. The causes are different one-sided limits, unbounded growth, or wild oscillation near the point.

Is the limit the same as the function value?

Not necessarily. The limit looks at nearby inputs, while the value is at the point itself. They agree exactly when the function is continuous there.

How is a hole different from a vertical asymptote?

A hole comes from a factor that cancels, so the limit is a finite number. An asymptote comes from a factor that does not cancel, so the function grows without bound.

When can I use the Squeeze Theorem?

Use it when the function is trapped between two simpler functions that approach the same limit at the same point, such as $x^2 \cos(7/x)$ near 0.

How do I find a horizontal asymptote?

Take the limit as x approaches ∞ and $-\infty$. For a rational function, compare the degrees: equal degrees give the ratio of leading coefficients, and a larger denominator degree gives $y = 0$.