

Limits and Continuity: Approaching a Value

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 1

Worked solutions and scoring guides follow. Award each point only for the stated reasoning, not just the final number. Limits must be written in correct notation, and a conclusion about continuity must name the failing condition.

Standards: AP Calculus AB Unit 1 · **Level:** Grade 12 AP Calculus AB

Question 1 Evaluate a limit (2 points)

Substitution gives $0/0$. Factor: $\frac{(x-2)(x+1)}{(x-2)(x+2)} = \frac{(x+1)}{(x+2)}$ near 2, which tends to $3/4$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point if the work shows the correct factorization but the choice is wrong.

Question 2 Reading a graph (3 points)

(a) Both branches approach 3 at $x = 2$, so the limit is **3**.

(b) $\lim_{x \rightarrow 4^-} k(x) = 1$, $\lim_{x \rightarrow 4^+} k(x) = 3$ and $k(4) = 1$.

(c) **Not continuous**: the one-sided limits at 4 are different, so the limit does not exist (jump discontinuity).

Scoring: 1 point per part.

Question 3 Limit at infinity (2 points)

Divide numerator and denominator by x^3 : $\frac{(7 - 1/x^2)}{(2/x^3 - 5)} \rightarrow 7/(-5)$. Answer **(A)**.

Scoring: 2 points for the correct choice; 1 point for noting that the degrees are equal.

Question 4 Conjugate method (2 points)

Multiply by the conjugate: $\frac{(x+16) - 16}{(x(\sqrt{x+16}) + 4)} = \frac{1}{(\sqrt{x+16}) + 4}$. At $x = 0$ this is $1/8$. The limit is **1/8**.

Scoring: 1 point for using the conjugate, 1 point for the correct simplification and value.

Question 5 Continuity at a point (3 points)

(a) Factor: $2x^2 - 11x - 6 = (2x + 1)(x - 6)$, so $f(x) = 2x + 1$ for $x \neq 6$, and the limit is 13.

(b) $f(6) = 10$ exists and the limit 13 exists, but $10 \neq 13$, so **not continuous**.

(c) Redefining $f(6) = \mathbf{13}$ removes the break, so the original is a **removable discontinuity**.

Scoring: 1 point per part.

Question 6 Discontinuities of a rational function (3 points)

(a) $f(x) = \frac{(x-3)(x+2)}{(x-3)(x+3)}$. At $x = 3$ the factor cancels and the limit is $5/6$: **removable**. At $x = -3$ the denominator factor remains: **infinite** (vertical asymptote).

(b) Near -3 the function behaves like $\frac{(x+2)}{(x+3)}$ with numerator close to -1 . From the left $x + 3 < 0$, so the limit is $+\infty$; from the right it is $-\infty$.

Scoring: Part (a): 1 point per correct classification with reason. Part (b): 1 point for both correct.

Question 7 Intermediate Value Theorem (2 points)

f is a polynomial, hence continuous on $[1, 2]$. $f(1) = 1 + 1 - 9 = -7$ and $f(2) = 16 + 2 - 9 = 9$. Since 0 is between -7 and 9 , the Intermediate Value Theorem gives a c in $(1, 2)$ with $f(c) = 0$. **A solution exists.**

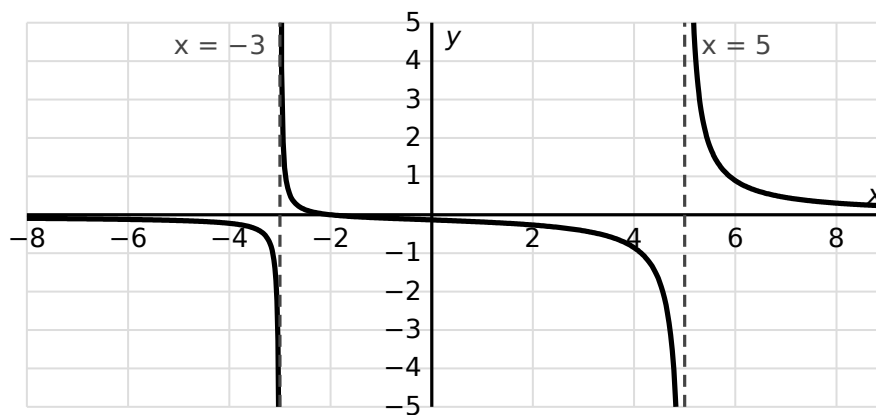
Scoring: 1 point for continuity, 1 point for the sign change and conclusion.

Question 8 Asymptotes and one-sided limits (3 points)

(a) For x slightly above 5: numerator $7 > 0$, $x - 5$ small positive, $x + 3 = 8 > 0$, so $+\infty$; from the left, $-\infty$.

(b) Numerator near -1 . From the left of -3 : $x + 3 < 0$, $x - 5 < 0$ (product of those two is positive small) so the quotient is negative: $-\infty$; from the right: $+\infty$.

(c) The degree of the denominator is greater, so the limit at $\pm\infty$ is 0 and the horizontal asymptote is **$y = 0$** .



Asymptotes at $x = -3$, $x = 5$ and $y = 0$.

Scoring: 1 point per part.