

Limits and Continuity: Approaching a Value

Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 1

Name: _____ Date: _____ Class: _____

These 20 problems build the skills of AP Calculus AB Unit 1: reading limits from graphs and tables, evaluating them with algebra, and linking them to continuity. The early problems use direct substitution and factoring; later ones bring in asymptotes and discontinuities.

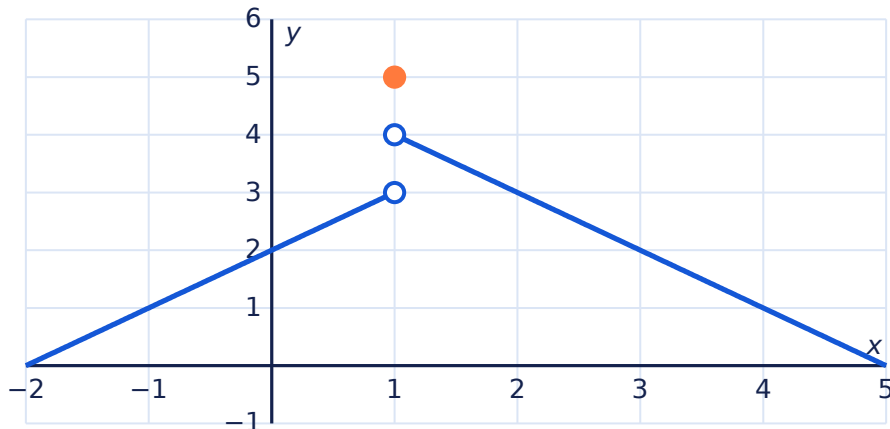
The last six problems are multi-step situations. They combine limits with the Intermediate Value Theorem, a garage fee schedule and a medicine model. Work each one fully before checking the answer key.

Standards: AP Calculus AB Unit 1 · **Level:** Grade 12 AP Calculus AB

Problem 1 Limits from a graph (easy)

The graph of h is shown. Find each value:

- a. $\lim_{x \rightarrow 1^-} h(x)$
- b. $\lim_{x \rightarrow 1^+} h(x)$
- c. $\lim_{x \rightarrow 1} h(x)$
- d. $h(1)$
- e. $\lim_{x \rightarrow 3} h(x)$



Graph of h .

Problem 2 Estimating from a table (easy)

A table of values for $f(x) = \frac{(x^3 - 8)}{(x - 2)}$ is given. Estimate $\lim_{x \rightarrow 2} f(x)$.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	11.41	11.9401	11.994	12.006	12.0601	12.61

Problem 3 Direct substitution (easy)

Evaluate $\lim_{x \rightarrow -2} (3x^2 - 4x + 1)$.

Problem 4 Factoring a 0/0 form (easy)

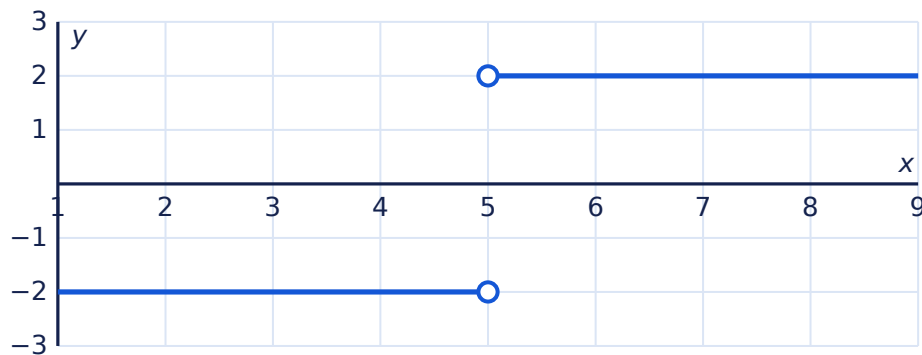
Evaluate $\lim_{x \rightarrow 4} \frac{(x^2 - x - 12)}{(x - 4)}$.

Problem 5 Using a conjugate (easy)

Evaluate $\lim_{x \rightarrow 0} \frac{(\sqrt{x+25} - 5)}{x}$.

Problem 6 One-sided limits with absolute value (medium)

Let $g(x) = \frac{(|2x - 10|)}{(x - 5)}$. Find $\lim_{x \rightarrow 5^-} g(x)$, $\lim_{x \rightarrow 5^+} g(x)$ and $\lim_{x \rightarrow 5} g(x)$.



Graph of g .

Problem 7 Limit at infinity of a rational function (medium)

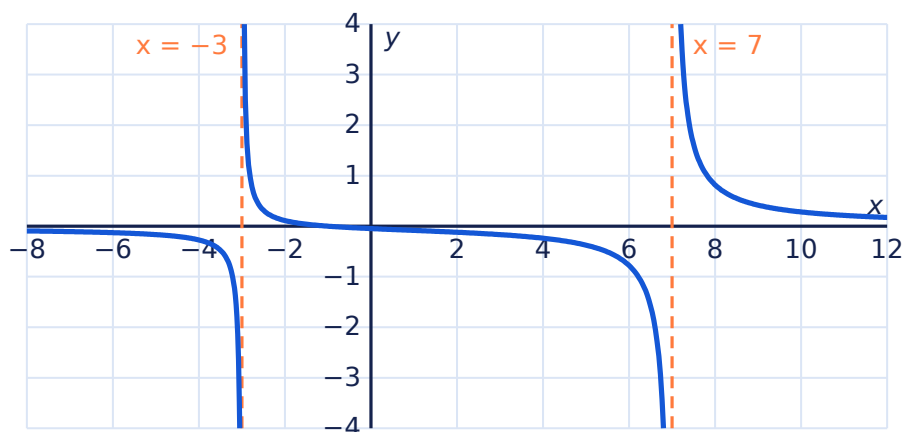
Evaluate $\lim_{x \rightarrow \infty} \frac{(6x^2 - x)}{(2x^2 + 5)}$ and state the horizontal asymptote.

Problem 8 Limits at infinity with a root (medium)

Evaluate $\lim_{x \rightarrow \infty} \frac{(4x + 1)}{(\sqrt{x^2 + 9})}$ and $\lim_{x \rightarrow -\infty} \frac{(4x + 1)}{(\sqrt{x^2 + 9})}$.

Problem 9 Infinite one-sided limits (medium)

Let $f(x) = \frac{(x+1)}{(x-7)(x+3)}$. Find $\lim_{x \rightarrow 7^+} f(x)$, $\lim_{x \rightarrow 7^-} f(x)$, and name every vertical asymptote.



Graph of f .

Problem 10 Squeeze Theorem (medium)

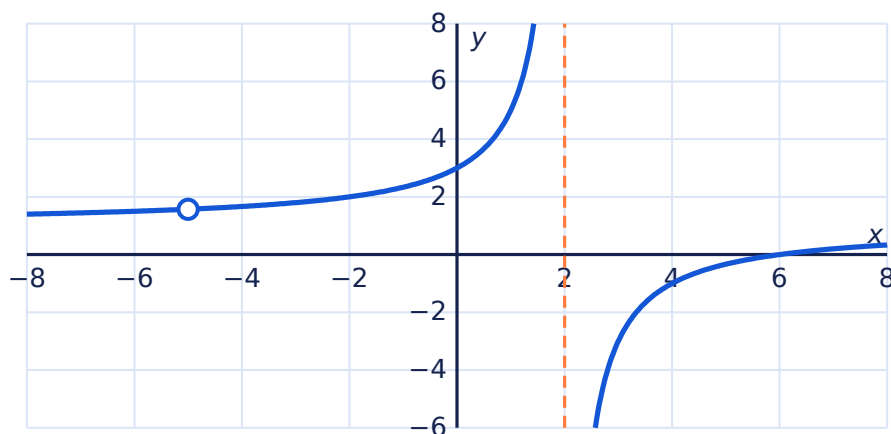
Use the Squeeze Theorem to evaluate $\lim_{x \rightarrow 0} x^2 \cos(7/x)$.

Problem 11 Making a function continuous (medium)

Let $f(x) = \frac{(x^2 - 2x - 35)}{(x - 7)}$ for $x \neq 7$ and $f(7) = k$. Find k so that f is continuous at $x = 7$.

Problem 12 Classifying discontinuities (medium)

Let $f(x) = \frac{((x+5)(x-6))}{((x+5)(x-2))}$. List the discontinuities, state their types and give the value or behavior of f near each.



Graph of f .

Problem 13 A piecewise function and one parameter (medium)

Let $f(x) = x^2 + ax$ for $x \leq 2$ and $f(x) = 7x - 9$ for $x > 2$. Find a so that f is continuous at $x = 2$.

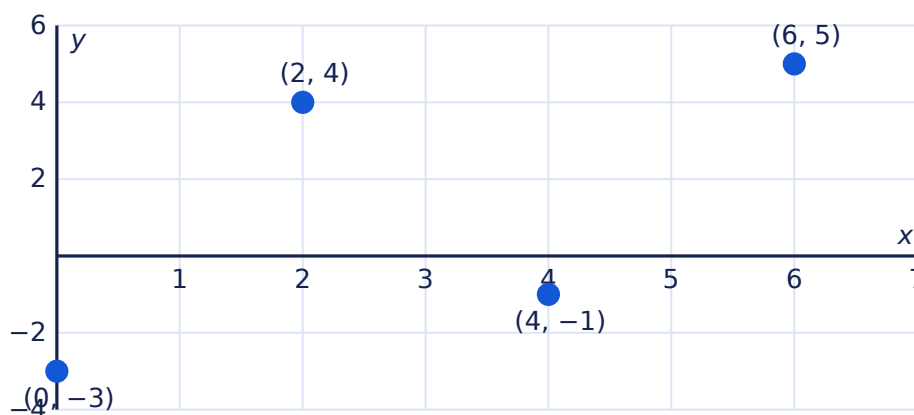
Problem 14 Intermediate Value Theorem for a polynomial (medium)

Show that $f(x) = x^3 + 3x - 11$ has a zero in the interval $(1, 2)$.

Problem 15 Counting guaranteed zeros (hard)

The function g is continuous on $[0, 6]$ and its known values are plotted.

- What is the least number of zeros of g in $(0, 6)$?
- Is $g(x) = 4.5$ guaranteed to have a solution in $[4, 6]$? Is it guaranteed in $[0, 2]$?

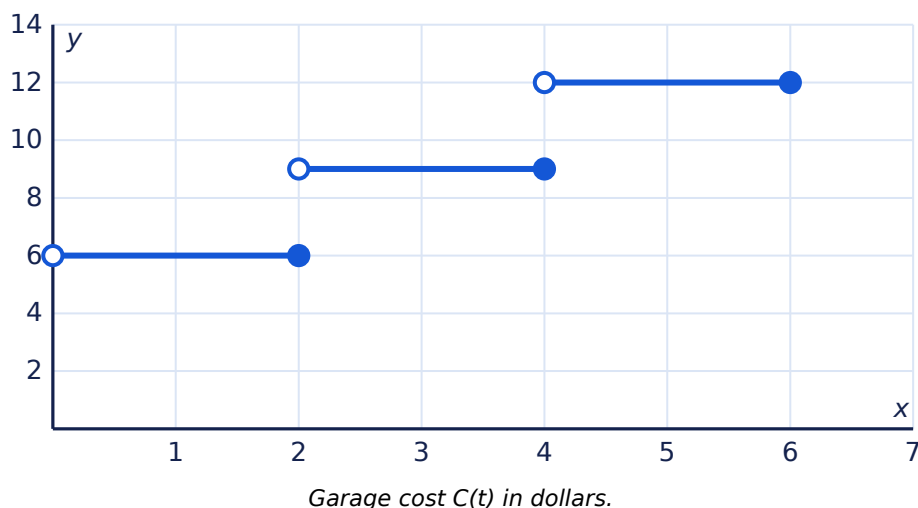


Known points of g .

Problem 16 Parking garage fees (hard)

A garage charges \$6 for a stay of up to 2 hours, \$9 for more than 2 and up to 4 hours, and \$12 for more than 4 and up to 6 hours. Let $C(t)$ be the cost for a stay of t hours.

- Find $\lim_{t \rightarrow 4^-} C(t)$ and $\lim_{t \rightarrow 4^+} C(t)$.
- Is C continuous at $t = 3$? At $t = 4$?
- What does a 4.5-hour stay cost?



Problem 17 Medicine level in the blood (hard)

The concentration of a medicine in a patient, in mg/L, is $C(t) = \frac{(45t + 20)}{(3t + 10)}$ where t is hours after the dose. Find $C(0)$, the limit as $t \rightarrow \infty$, and the time when the concentration reaches 14 mg/L.

Problem 18 Repairing a cubic quotient (hard)

Let $f(x) = \frac{(x^3 - 27)}{(x^2 - 9)}$. Find every discontinuity, classify it, and tell how to redefine f to remove the removable one.

Problem 19 Horizontal and vertical asymptotes with a root (hard)

Let $f(x) = \frac{(\sqrt{9x^2 + 4x})}{(x + 1)}$. Find its horizontal asymptotes and describe its behavior at $x = -1$.

Problem 20 Continuity of a three-piece function (hard)

Let $f(x) = 2x + a$ for $x < -1$; $bx + 3$ for $-1 \leq x \leq 2$; and $x^2 + 1$ for $x > 2$. Find a and b so that f is continuous for all real x .