

Motion, Related Rates and Linearization

Grade 12 – AP Calculus AB math · Test Answer Key · Standards: AP Calculus AB Unit 4

Worked solutions with scoring guides follow. Give credit for a correct setup and differentiation even when a later arithmetic slip occurs, but withhold the units point when units are missing in context answers.

Standards: AP Calculus AB Unit 4 · **Level:** Grade 12 AP Calculus AB

Question 1 Direction change (2 points)

$v(t) = 3t^2 - 12 = 3(t - 2)(t + 2)$. For $t \geq 0$ the velocity changes from negative to positive at $t = 2$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for finding $v(t) = 0$.

Question 2 L'Hôpital multiple choice (2 points)

The form is $0/0$. First pass: $(e^x - 1)/2x$, still $0/0$. Second pass: $e^x/2 \rightarrow 1/2$. Answer **(B)**.

Scoring: 2 points for the correct choice; 1 point for a correct first application.

Question 3 Rectangle related rates (3 points)

(a) $A = lw$, so $dA/dt = l'w + lw' = 3(6) + 10(-2) = -2 \text{ in}^2/\text{s}$.

(b) The rate is negative, so the area is **decreasing**.

(c) $D^2 = l^2 + w^2$ gives $D = \sqrt{136} = 2\sqrt{34}$ and $dD/dt = (ll' + ww')/D = (30 - 12)/(2\sqrt{34}) = 9/\sqrt{34} \approx 1.54 \text{ in/s}$.

Scoring: 1 point per part.

Question 4 Linearization of a square root (3 points)

(a) $f(49) = 7$ and $f'(x) = 1/(2\sqrt{x})$, so $f'(49) = 1/14$. $L(x) = 7 + (x - 49)/14$.

(b) $\sqrt{50} \approx L(50) = 7 + 1/14 \approx 7.0714$.

(c) $f'' < 0$, so f is concave down and the tangent line lies above the graph. The approximation is

greater than $\sqrt{50}$.

Scoring: 1 point per part.

Question 5 Velocity graph analysis (3 points)

(a) $v = 0$ at $t = 4$.

(b) On $(4, 6)$ the graph goes from 0 to -3 in 2 seconds, so $a(5) = -3/2 \text{ m/s}^2$.

(c) $v(5) < 0$ and $a(5) < 0$, the same sign, so the particle is **speeding up**.



The segment from $(4, 0)$ to $(6, -3)$ has slope $-3/2$.

Scoring: 1 point per part.

Question 6 Marginal analysis (3 points)

(a) $C'(x) = 0.02x + 5$.

(b) $C'(200) = 9$. When 200 backpacks are made, cost is rising at **\$9 per backpack**.

(c) $C(201) - C(200) = 0.01(40,401 - 40,000) + 5 = \mathbf{\$9.01}$, very close to the estimate of \$9.

Scoring: 1 point per part.

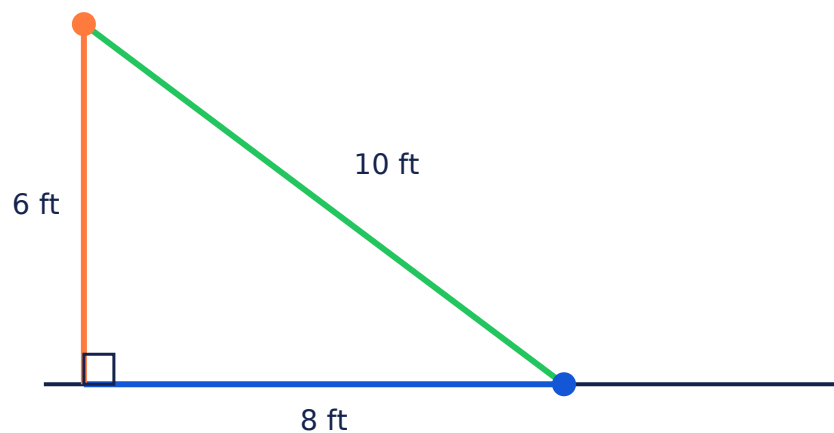
Question 7 L'Hôpital with a logarithm (2 points)

The form is ∞/∞ . Differentiate: $\frac{1/x}{1/(2\sqrt{x})} = \frac{2}{\sqrt{x}}$, which tends to 0. The limit is **0**.

Scoring: 1 point for naming the form, 1 point for the derivative quotient and the limit.

Question 8 Sliding ladder test (2 points)

$x^2 + y^2 = 100$. When $y = 6$, $x = 8$. Differentiate: $x \frac{dx}{dt} + y \frac{dy}{dt} = 0$, so $8 \frac{dx}{dt} + 6(-1.2) = 0$ and $\frac{dx}{dt} = 7.2/8 = \mathbf{0.9 \text{ ft/s}}$.



The base is 8 feet from the wall.

Scoring: 1 point for the correct equation and derivative, 1 point for the value 0.9.