

Motion, Related Rates and Linearization

Grade 12 – AP Calculus AB math · Test · Standards: AP Calculus AB Unit 4

Name: _____ Date: _____ Class: _____

This 45-minute test follows the AP Calculus AB style, with two multiple-choice items and free-response questions requiring units and justification. Show every step. The total is 20 points, and no calculator is needed.

Standards: AP Calculus AB Unit 4 · **Level:** Grade 12 AP Calculus AB

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

Question 1 Direction change (2 points)

A particle moves along a line with position $s(t) = t^3 - 12t + 5$ for $t \geq 0$. At what time does the particle change direction?

- A. 1
- B. 2
- C. 4
- D. 6

Question 2 L'Hôpital multiple choice (2 points)

What is $\lim_{x \rightarrow 0} \frac{(e^x - 1 - x)}{x^2}$?

- A. 0
- B. 1/2
- C. 1
- D. The limit does not exist

Question 3 Rectangle related rates (3 points)

The length of a rectangle increases at 3 in/s while its width decreases at 2 in/s. At the moment the length is 10 in and the width is 6 in:

- a. Find the rate of change of the area. (1 point)
- b. Is the area increasing or decreasing? Justify. (1 point)
- c. Find the rate of change of the diagonal. (1 point)

Question 4 Linearization of a square root (3 points)

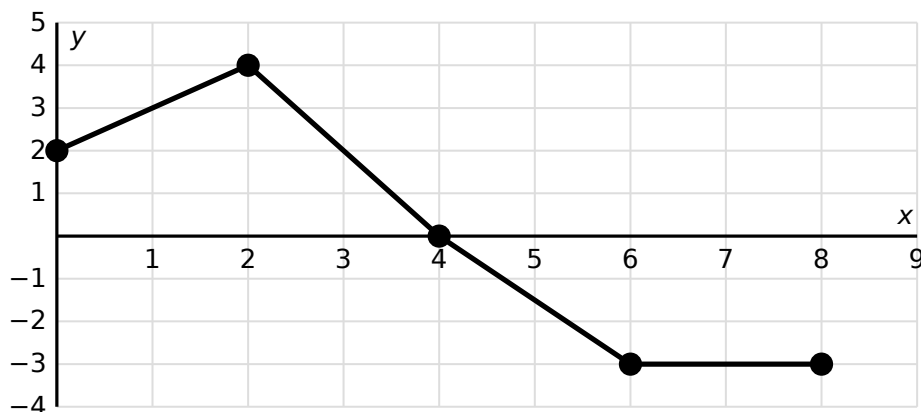
Let $f(x) = \sqrt{x}$.

- Write the equation of the tangent line to f at $x = 49$. (1 point)
- Use it to approximate $\sqrt{50}$. (1 point)
- Is your approximation greater or less than $\sqrt{50}$? Explain. (1 point)

Question 5 Velocity graph analysis (3 points)

The velocity $v(t)$ of a particle, in m/s, is graphed for $0 \leq t \leq 8$. It consists of line segments.

- At what time is the particle momentarily at rest? (1 point)
- Find the acceleration at $t = 5$. (1 point)
- At $t = 5$ is the particle speeding up or slowing down? Explain. (1 point)



Velocity of the particle.

Question 6 Marginal analysis (3 points)

The cost in dollars of making x backpacks is $C(x) = 0.01x^2 + 5x + 300$.

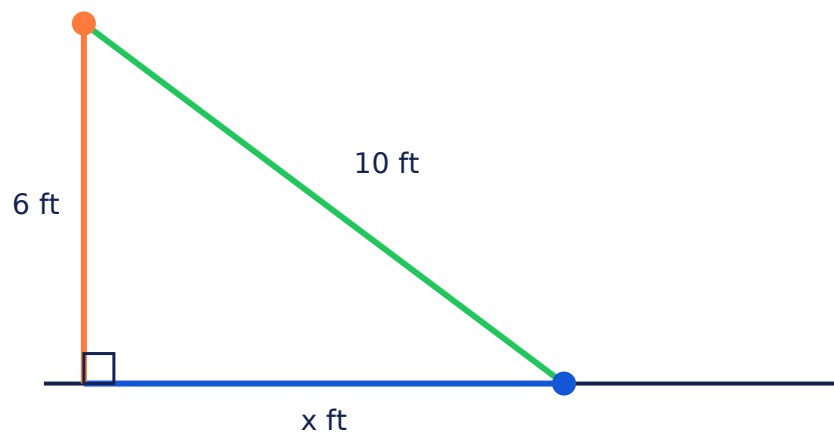
- Find $C'(x)$. (1 point)
- Find $C'(200)$ and interpret it with units. (1 point)
- Compute $C(201) - C(200)$ and compare. (1 point)

Question 7 L'Hôpital with a logarithm (2 points)

Evaluate $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{\sqrt{x}}$ and show the form that justifies your method.

Question 8 Sliding ladder test (2 points)

A 10-foot ladder leans against a wall. The top slides down at 1.2 ft/s. How fast is the base moving away from the wall when the top is 6 feet above the ground?



The ladder at the moment in question.