

# Motion, Related Rates and Linearization

## Grade 12 – AP Calculus AB math · Worksheet · Standards: AP Calculus AB Unit 4

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

These 20 problems cover AP Calculus AB Unit 4: interpreting derivatives with units, position, velocity and acceleration, related rates, local linearization and L'Hôpital's Rule. Early problems build vocabulary; later ones ask for a full related-rates setup.

The last six problems are multi-step situations such as a walking shadow, a radar station and a particle that changes direction.

**Standards:** AP Calculus AB Unit 4 · **Level:** Grade 12 AP Calculus AB

### Problem 1 Interpreting marginal cost (easy)

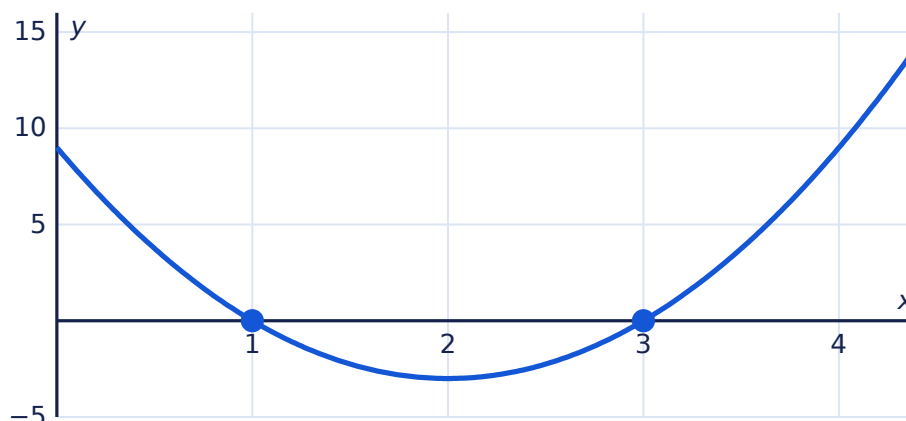
The cost, in dollars, of baking  $q$  cupcakes is  $C(q)$ , and  $C'(200) = 1.85$ . Explain what this value means in context, with units.

### Problem 2 Velocity and acceleration at an instant (easy)

A cart's position in feet is  $s(t) = t^3 - 9t^2 + 15t$ . Find its velocity, acceleration and speed at  $t = 2$ .

### Problem 3 Direction of motion (medium)

A particle on a line has position  $s(t) = t^3 - 6t^2 + 9t$  for  $t \geq 0$ . Find when it moves right and when it moves left.



#### Problem 4 Acceleration from a table (medium)

The velocity  $v(t)$  of a robot, in meters per second, is recorded. Estimate its acceleration at  $t = 3.5$  using the two nearest times, with units.

| $t$ (s) | 0 | 2 | 5  | 9  |
|---------|---|---|----|----|
| $v(t)$  | 3 | 7 | 16 | 20 |

#### Problem 5 Speeding up or slowing down (medium)

A particle has velocity  $v(t) = t^2 - 8t + 12$ . Decide whether it is speeding up or slowing down at  $t = 1$  and at  $t = 7$ .

#### Problem 6 Linearization of a cube root (medium)

Write the local linearization of  $f(x) = \sqrt[3]{x}$  at  $x = 27$  and use it to approximate  $\sqrt[3]{29}$ . Is the estimate too high or too low?

#### Problem 7 Linear approximation of an exponential (medium)

Use the tangent line to  $g(x) = e^{5x}$  at  $x = 0$  to approximate  $g(0.02)$ . Is the approximation an underestimate or an overestimate?

#### Problem 8 L'Hôpital's Rule once (medium)

Evaluate  $\lim_{x \rightarrow 0} \frac{(e^{4x} - 1)}{(\sin 6x)}$ .

#### Problem 9 L'Hôpital's Rule twice (medium)

Evaluate  $\lim_{x \rightarrow \infty} \frac{x^2}{e^{3x}}$ .

#### Problem 10 Growing circle (medium)

A circular oil spill's radius grows at 0.4 cm/s. How fast is the area increasing when the radius is 15 cm?

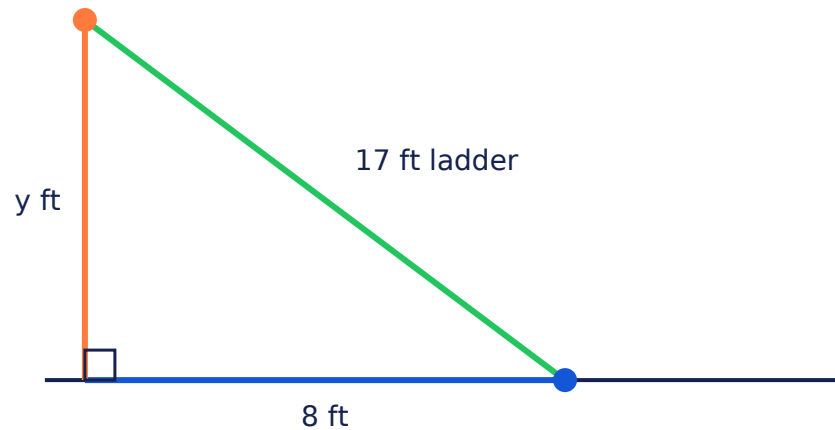
#### Problem 11 Inflating a balloon (medium)

A spherical balloon is filled with air at  $90 \text{ in}^3/\text{s}$ . How fast is the radius increasing when the radius

is 3 inches? (Volume of a sphere:  $V = (4/3)\pi r^3$ .)

### Problem 12 Sliding ladder (hard)

A 17-foot ladder leans against a wall. Its base slides away at 1.5 ft/s. How fast is the top sliding down the wall when the base is 8 feet from the wall?



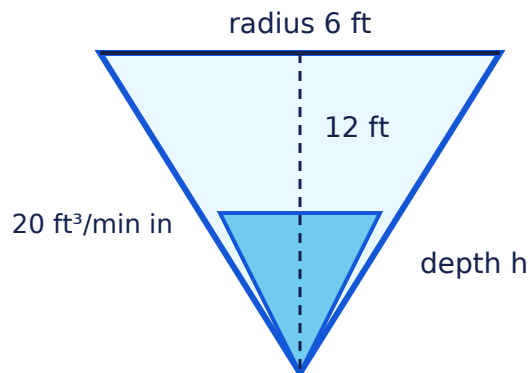
*The ladder at the moment in question.*

### Problem 13 Marginal cost and actual cost (hard)

The cost, in dollars, of producing  $x$  skateboards is  $C(x) = 0.02x^2 + 4x + 500$ . Find  $C'(100)$ , interpret it, and compare it with the actual cost of the 101st skateboard.

### Problem 14 Filling a conical tank (hard)

A tank shaped like an upside-down cone has radius 6 feet and height 12 feet. Water flows in at  $20 \text{ ft}^3/\text{min}$ . How fast is the depth rising when the depth is 4 feet?



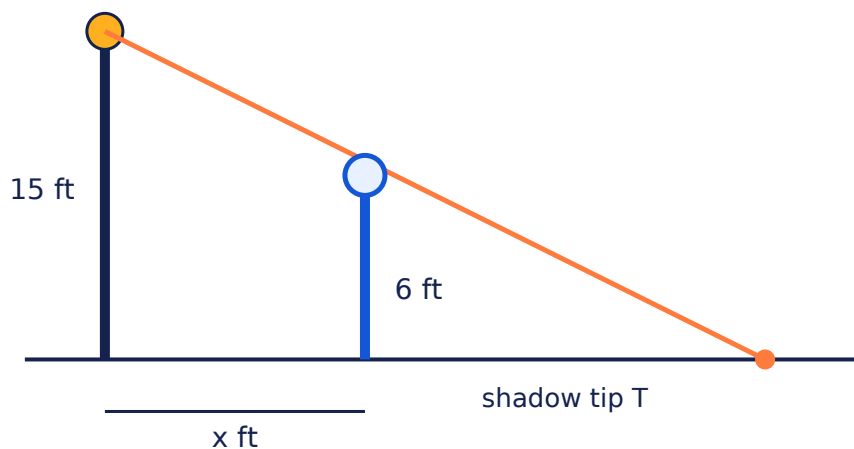
*The conical tank.*

**Problem 15 Relative growth of a population** (hard)

A town's population is  $P(t) = 8,000e^{0.04t}$ , with  $t$  in years. Find  $P'(t)$ , the rate of change at  $t = 5$ , and the relative rate  $P'/P$ .

**Problem 16 Shadow of a person walking** (hard)

A 6-foot person walks away from a 15-foot lamp post at 4 ft/s. How fast is the tip of the shadow moving along the ground, and how fast is the shadow's length growing?



Lamp post, person and shadow.

**Problem 17 Distance traveled by a particle** (hard)

A particle's position is  $s(t) = t^3 - 12t^2 + 36t$  for  $0 \leq t \leq 7$ . Find the total distance traveled and the net displacement.

**Problem 18 Trigonometric limit with two passes** (hard)

Evaluate  $\lim_{x \rightarrow 0} \frac{(1 - \cos 5x)}{x^2}$  and name each form that appears.

**Problem 19 Intervals of speeding up** (hard)

A particle's position is  $s(t) = 2t^3 - 15t^2 + 36t$  for  $0 \leq t \leq 4$ . On which intervals is the particle speeding up?

**Problem 20 Distance from a radar station** (hard)

A plane flies horizontally at an altitude of 4 miles and a speed of 500 mph, moving directly away from a radar station. How fast is the distance from the station to the plane increasing when the

plane is 3 miles of ground distance from the station?

