

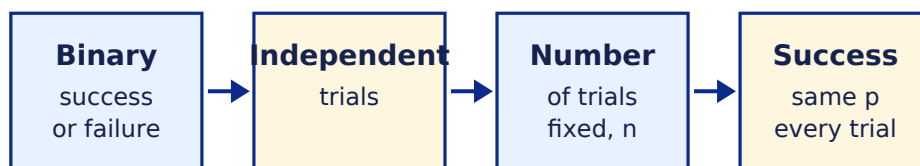
Binomial and Normal Probability Models

Grade 12 – AP Statistics math · Lesson · Standards: AP Statistics Unit 2 (2026-27)

Quick answer: A binomial random variable counts successes in a fixed number of independent trials with the same success probability, with mean np and standard deviation $\sqrt{np(1 - p)}$. Normal distributions model continuous bell-shaped data and, when np and $n(1 - p)$ are at least 10, approximate binomial counts.

Standards: AP Statistics Unit 2 (2026-27) · **Level:** Grade 12 AP Statistics

What makes a setting binomial?



BINS: the four conditions of a binomial setting.

Binomial setting (BINS)

A count X of successes is binomial when each trial has two outcomes, trials are independent, the number of trials n is fixed, and the probability of success p is the same on every trial. Write $X \sim B(n, p)$.

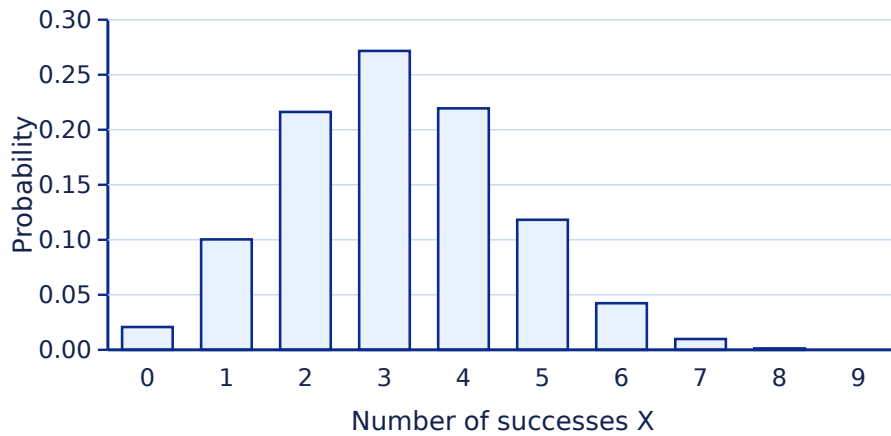
Sampling without replacement is treated as independent when the sample is less than 10% of the population.

How do you find binomial probabilities, the mean and the standard deviation?

Binomial formulas

$$P(X = k) = C(n, k) p^k (1 - p)^{n - k}$$

$$\mu_x = np \quad \sigma_x = \sqrt{np(1 - p)}$$



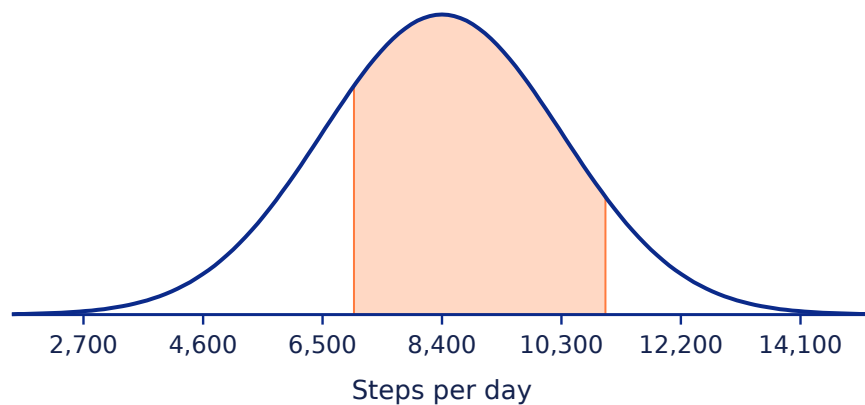
Binomial distribution with $n = 9$ and $p = 0.35$.

Worked example

Each seed germinates with probability 0.35, and 9 seeds are planted. Then $P(X = 3) = C(9, 3)(0.35)^3(0.65)^6 = 0.2716$, and $P(X \leq 2) = 0.3373$. The mean is $9(0.35) = 3.15$ and $\sigma = \sqrt{9 \times 0.35 \times 0.65} \approx 1.431$.

How do you work with normal models?

A normal distribution is symmetric and bell-shaped, determined by μ and σ . Standardize with $z = (x - \mu)/\sigma$, then find the area with a table or calculator (*normalcdf*), or use *invNorm* to go from an area back to a value.



Daily steps; shaded area is between 7,000 and 11,000.

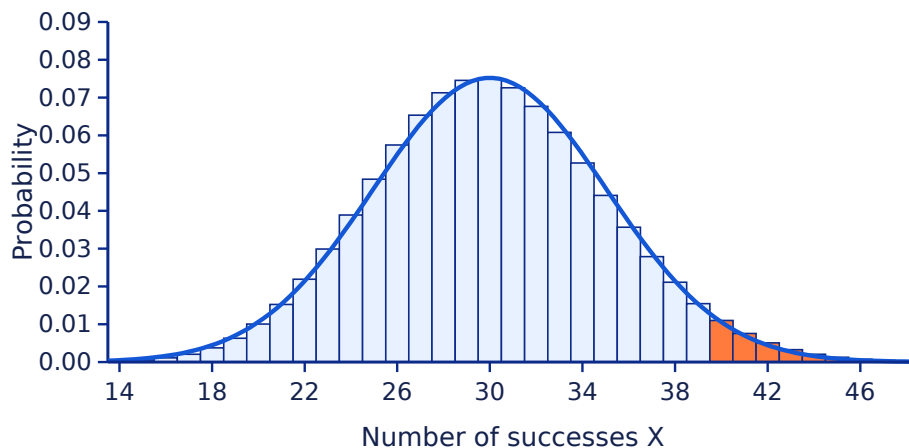
Daily steps are normal with mean 8,400 and standard deviation 1,900. For 7,000 and 11,000: $z = -0.74$ and 1.37 , so the area between them is $0.9144 - 0.2306 = 0.6838$.

When can a normal curve approximate a binomial count?

Normal approximation to $B(n, p)$

1. Check $np \geq 10$ and $n(1 - p) \geq 10$.

2. Use $\mu = np$ and $\sigma = \sqrt{np(1 - p)}$.
3. Standardize and find the area for the event.



The normal curve with $\mu = 30$ and $\sigma \approx 5.05$ fits the binomial bars.

For $n = 200$ and $p = 0.15$, $np = 30$ and $n(1 - p) = 170$ pass the check. Then $P(X \geq 40) \approx P(Z \geq 1.98) = 0.0238$, close to the exact binomial 0.0335.

Mathbot's tip

A binomial count has a fixed number of trials. If you keep going until the first success, the number of trials is not fixed and the setting is not binomial.

Key points to remember

- Binomial setting: BINS (binary, independent, number fixed, same p).
- $P(X = k) = C(n, k) \cdot p^k \cdot (1 - p)^{n - k}$; mean np ; SD $\sqrt{np(1 - p)}$.
- Normal probabilities come from $z = (x - \mu)/\sigma$ and the area under the curve.
- Use `invNorm` to find a value from a percentile.
- Normal approximation to a binomial needs $np \geq 10$ and $n(1 - p) \geq 10$.

Binomial and Normal Probability Models: frequently asked questions

How do I check whether a situation is binomial?

Use BINS: two outcomes per trial, independent trials, a fixed number of trials, and the same success probability each time. If any condition fails, such as sampling without replacement from a small population, do not use the binomial formula.

What is the difference between `binompdf` and `binomcdf`?

`binompdf(n, p, k)` gives $P(X = k)$, a single probability. `binomcdf(n, p, k)` gives $P(X \leq k)$, the cumulative probability. For $P(X \geq k)$, compute $1 - \text{binomcdf}(n, p, k - 1)$.

When is the normal approximation to a binomial allowed?

When both np and $n(1 - p)$ are at least 10. Then the binomial distribution is roughly symmetric and bell-shaped, and a normal model with mean np and standard deviation $\sqrt{np(1 - p)}$ gives a good estimate.

How do I find a value from a percentile in a normal model?

Use $\text{invNorm}(\text{area}, \mu, \sigma)$, or find the z-score for the area and compute $x = \mu + z\sigma$. For the top 10%, the area to the left is 0.90.

Does a normal distribution have to be exactly symmetric to use the empirical rule?

The model itself is symmetric. Real data are only approximately normal, so check the histogram or a normal probability plot first. Skewed data should not be modeled by a normal distribution.