

Binomial and Normal Probability Models

Grade 12 – AP Statistics math · Worksheet Answer Key · Standards: AP Statistics Unit 2 (2026-27)

The answer key shows the BINS check, the formula or calculator command, the standardizing step and the final probability. Probabilities are rounded to four decimal places during the work and to three in the final answer.

Standards: AP Statistics Unit 2 (2026-27) · **Level:** Grade 12 AP Statistics

Problem 1 Is It Binomial?

(a) Two outcomes, independent trials, fixed $n = 15$, constant p : binomial. (b) The number of trials is not fixed (cards are dealt until the first heart): not binomial. (c) The sample is 5 of 28, more than 10% of the class, and sampling without replacement changes the probability: independence fails, so not binomial.

(a) binomial; (b) not binomial, number of trials not fixed; (c) not binomial, independence fails

Problem 2 A Binomial Probability by Formula

$$P(X = 2) = C(6, 2)(0.4)^2(0.6)^4 = 15 \times 0.16 \times 0.1296 = 0.3110.$$

$$\approx 0.3110$$

Problem 3 A Cumulative Binomial Probability

$$P(0) = 0.75^8 = 0.1001. P(1) = 8(0.25)(0.75)^7 = 0.2670. \text{ The sum is } 0.3671.$$

$$\approx 0.3671$$

Problem 4 Mean and Standard Deviation of a Count

$$\mu = 60(0.35) = 21. \sigma = \sqrt{60 \times 0.35 \times 0.65} = \sqrt{13.65} = 3.695.$$

$$\mu = 21, \sigma \approx 3.69$$

Problem 5 Reading a Binomial Histogram

The tallest bar is at 5 wins (0.2787). $P(X \geq 7) = P(7) + P(8) = 0.0896 + 0.0168 = 0.1064$. The mean is $8(0.6) = 4.8$.

5 wins; 0.1064; mean 4.8

Problem 6 At Least One Success

Let a failure to sprout have probability 0.15. $P(\text{at least one fails}) = 1 - P(\text{all sprout}) = 1 - 0.85^5 = 1 - 0.4437 = 0.5563$.

≈ 0.5563

Problem 7 Binomial with a Calculator

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - \text{binomcdf}(15, 0.3, 6) = 1 - 0.8689 = 0.1311$.

≈ 0.1311

Problem 8 Is the Result Unusual?

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.9568 = 0.0432$. A probability of about 4.3% is small but not extremely small, so the result is somewhat unusual yet could occur by chance.

0.0432; fairly unlikely, though not impossible

Problem 9 Normal Area to the Left

$z = (12 - 14.5)/2.1 = -1.19$. The area to the left is 0.1169.

≈ 0.1169

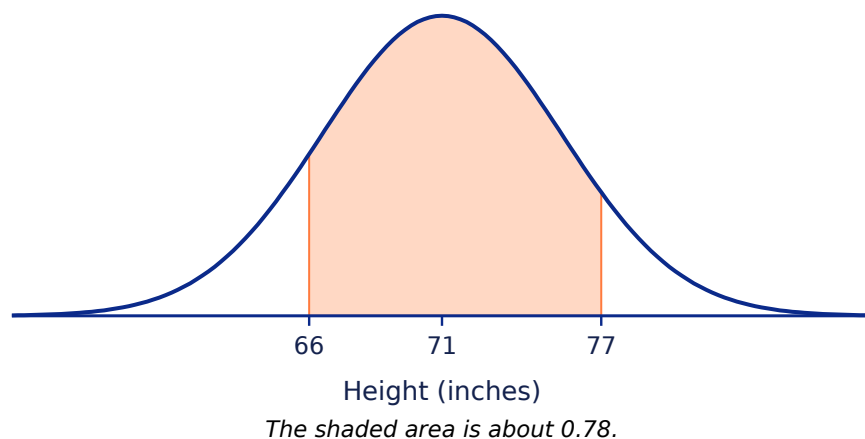
Problem 10 Normal Area to the Right

$z = (18 - 14.5)/2.1 = 1.67$. The area to the right is $1 - 0.9522 = 0.0478$.

≈ 0.0478

Problem 11 Normal Area Between Two Values

$z = (66 - 71)/4.5 = -1.11$ and $z = (77 - 71)/4.5 = 1.33$. Area = $0.9088 - 0.1333 = 0.7755$.



≈ 0.7755

Problem 12 Finding a Cutoff

The cutoff has 90% of scores below it, so $z = 1.282$ and $x = 78 + 1.282(8) = 88.25$.

about 88.3 points

Problem 13 Quartiles of a Normal Model

For the 25th and 75th percentiles, $z = \pm 0.6745$. $Q_1 = 24 - 0.6745(6.5) = 19.62$; $Q_3 = 24 + 0.6745(6.5) = 28.38$.

$Q_1 \approx \$19.62$, $Q_3 \approx \$28.38$

Problem 14 Comparing Two Normal Models

Reading: $z = (672 - 640)/24 = 1.33$, percentile 90.9. Math: $z = (58 - 52)/5 = 1.20$, percentile 88.5.

Reading: about the 91st percentile, higher than math at about the 88th

Problem 15 Binomial Probability in Context

$P(X \leq 2) = 0.6127 + 0.3064 + 0.0702 = 0.9893$. The expected number is $12(0.04) = 0.48$.

0.9893; 0.48 defective batteries per box

Problem 16 Surprising Counts

$\mu = 50(0.3) = 15$, $\sigma = \sqrt{(50 \times 0.3 \times 0.7)} = 3.240$. $z = (25 - 15)/3.240 = 3.09$. Exact: $P(X \geq 25)$

$= 1 - \text{binomcdf}(50, 0.3, 24) = 0.0024$. A result this far above the mean is very unlikely if the claim is true.

$\mu = 15$, $\sigma \approx 3.24$; $z \approx 3.09$; $P \approx 0.0024$

Problem 17 Normal Approximation Setup

Check: $np = 60 \geq 10$ and $n(1 - p) = 90 \geq 10$. $\mu = 60$, $\sigma = \sqrt{(150 \times 0.4 \times 0.6)} = 6.000$. $z = (70 - 60)/6.000 = 1.67$, so $P(X \geq 70) \approx 0.0478$. (The exact binomial probability is 0.0575.)

≈ 0.048

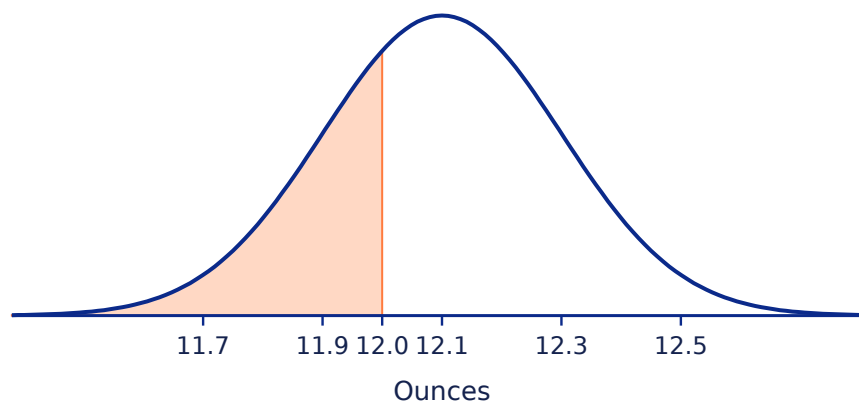
Problem 18 Approximating a Lower Tail

$np = 48 \geq 10$ and $n(1 - p) = 352 \geq 10$. $\mu = 48$, $\sigma = \sqrt{(400 \times 0.12 \times 0.88)} = 6.499$. $z = (40 - 48)/6.499 = -1.23$, so the area to the left is 0.1092.

≈ 0.109

Problem 19 Normal Model Then Binomial

(a) $z = (12 - 12.1)/0.2 = -0.50$, so $p = 0.3085$. (b) $P(\text{none underfilled}) = (1 - 0.3085)^8 = 0.0523$, so $P(\text{at least one}) = 0.9477$.



Underfilled cups: the area below 12 ounces.

(a) 0.3085; (b) 0.9477

Problem 20 Finding μ and σ from Two Percentiles

For 10% below, $z = -1.282$, so $40 = \mu - 1.282\sigma$. For 5% above, $z = 1.645$, so $70 = \mu + 1.645\sigma$. Subtract: $30 = 2.9264\sigma$, so $\sigma = 10.25$. Then $\mu = 40 + 1.282(10.25) = 53.14$.

$\mu \approx 53.1$ minutes, $\sigma \approx 10.3$ minutes