

Chi-Square Tests for Two-Way Tables

Grade 12 – AP Statistics math · Worksheet Answer Key · Standards: AP Statistics Unit 3 (2026-27)

The answer key shows the expected counts, the chi-square statistic and the right-tail p-value for each table, and gives a conclusion in context. Statistics were computed with exact cell contributions before rounding.

Standards: AP Statistics Unit 3 (2026-27) · **Level:** Grade 12 AP Statistics

Problem 1 Homogeneity or Independence

(a) One sample, two categorical variables: independence. (b) Separate samples from three populations, one categorical variable: homogeneity. (c) Randomly assigned treatment groups, one categorical outcome: homogeneity.

(a) independence; (b) homogeneity; (c) homogeneity

Problem 2 One Expected Count

Expected = $(85)(120)/340 = 30.0$.

30.0

Problem 3 An Expected Count Table

Row totals are 60 and 110, column totals are 80 and 90, and the table total is 170.

	Passed	Failed
Used app	28.24	31.76
No app	51.76	58.24

For example, $(60)(80)/170 = 28.235$.

28.24, 31.76, 51.76, 58.24

Problem 4 Degrees of Freedom

$df = (\text{rows} - 1)(\text{columns} - 1)$. (a) $(2)(3) = 6$. (b) $(1)(1) = 1$. (c) $(4)(2) = 8$.

(a) 6; (b) 1; (c) 8

Problem 5 A Cell's Contribution

$$(O - E)^2/E = (41 - 33.2)^2/33.2 = 60.84/33.2 = 1.833.$$

$$\approx 1.833$$

Problem 6 The Statistic for a 2 by 2 Table

Cell contributions: 1.177, 1.046, 0.642, 0.571. The sum is $\chi^2 = 3.436$, $df = 1$, $p\text{-value} = 0.0638$.

$$\chi^2 \approx 3.44, df = 1, p\text{-value} \approx 0.064$$

Problem 7 Finding a p-Value

$p\text{-value} = P(\chi^2 \geq 9.35) = 0.0250$, from a calculator ($\chi^2\text{cdf}$ with lower bound 9.35). Since $0.0250 < 0.05$, reject H_0 .

p-value ≈ 0.0250 ; reject H_0

Problem 8 Checking the Expected Counts

Row totals 30 and 70; column totals 13, 17 and 70; table total 100.

	Yes	Maybe	No
Group 1	3.90	5.10	21.00
Group 2	9.10	11.90	49.00

The expected count 3.9 is below 5, so the condition fails.

Not appropriate: expected count $3.9 < 5$

Problem 9 Writing Hypotheses

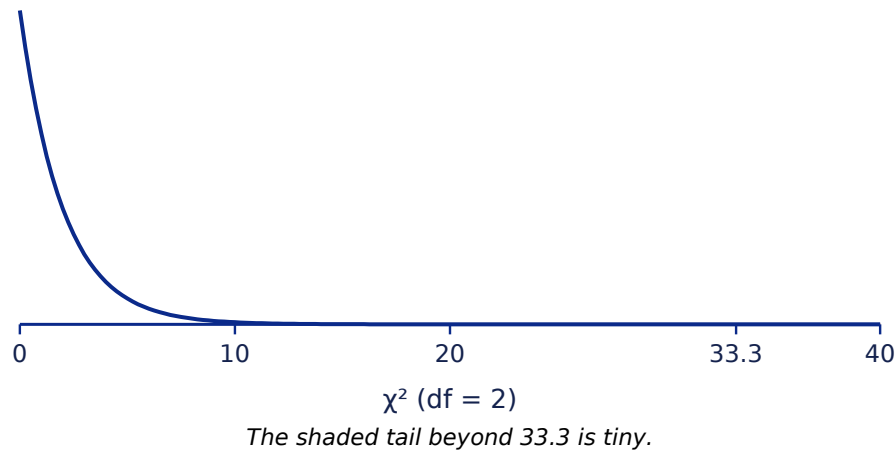
(a) H_0 : the distribution of commute method is the same in all three cities. H_a : the distribution of commute method is not the same in all three cities. (b) H_0 : age group and preferred candidate are independent in the population of voters. H_a : age group and preferred candidate are associated.

(a) same distribution vs not; (b) independent vs associated

Problem 10 A Complete Test of Independence

State: H_0 : age group and payment method are independent for these shoppers; H_a : they are

associated. **Plan:** chi-square test for independence; random sample, 250 less than 10% of all shoppers, expected counts are 62.4, 26.0, 41.6, 57.6, 24.0, 38.4, all at least 5. **Do:** $\chi^2 = 33.31$, $df = 2$, $p\text{-value} < 0.0001$. **Conclude:** the $p\text{-value}$ is below 0.05, so there is convincing evidence that age group and payment method are associated.



$\chi^2 \approx 33.3$, $df = 2$, $p\text{-value} < 0.0001$; reject H_0

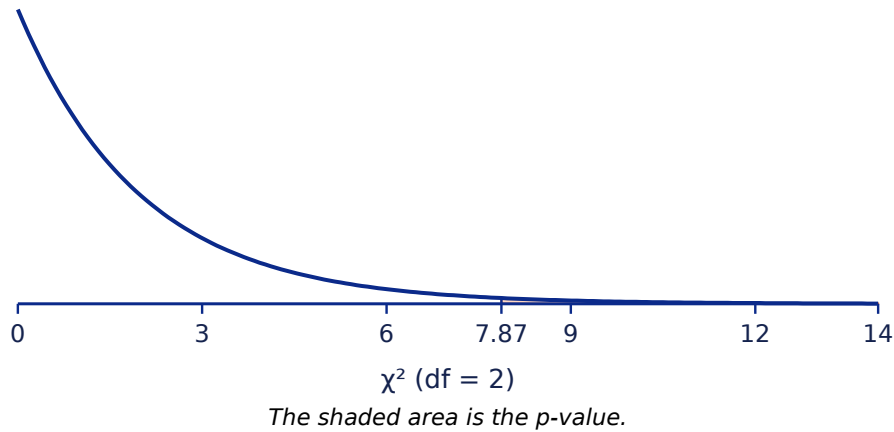
Problem 11 Reading a Segmented Bar Graph

The percent choosing hot lunch falls from 41% at Lincoln to 20% at Jefferson, and the percent skipping lunch rises from 25% to 39%. The distributions look different, but a test is needed to see whether the differences could be due to sampling variability. Use a chi-square test for homogeneity (three samples, one categorical variable).

Distributions appear different; chi-square test for homogeneity

Problem 12 A Test of Homogeneity

H_0 : the distribution of passing and failing is the same for the three methods; H_a : it is not.
 Conditions: random assignment; expected counts are 34.33, 15.67, 34.33, 15.67, 34.33, 15.67, all at least 5. $\chi^2 = 7.870$, $df = 2$, $p\text{-value} = 0.0195$. Since $0.0195 < 0.05$, reject H_0 : there is convincing evidence that the pass rates differ among the methods, and because students were randomly assigned, the method plausibly causes the difference.



$\chi^2 \approx 7.87$, df = 2, p-value ≈ 0.0195 ; reject H_0

Problem 13 Interpreting a p-Value

If vehicle type and region were independent in the population, the probability of getting a sample with an association at least as strong as the one observed, just by random sampling, would be 0.027.

Assuming independence, a result this extreme has probability 0.027

Problem 14 Decisions at Two Levels

p-value = 0.0197. At $\alpha = 0.05$, $0.0197 < 0.05$: reject H_0 . At $\alpha = 0.01$, $0.0197 > 0.01$: fail to reject H_0 .

Reject at 0.05; fail to reject at 0.01

Problem 15 Which Cells Differ Most?

Contributions $((O - E)^2/E)$: Under 35 Card: 0.31, Under 35 Cash: 7.54, Under 35 Phone app: 8.14, 35 and over Card: 0.34, 35 and over Cash: 8.17, 35 and over Phone app: 8.82. The largest are for ages 35 and over using the phone app (observed 20, expected 38.4) and ages 35 and over using cash (observed 38, expected 24.0).

35 and over, phone app (fewer than expected) and 35 and over, cash (more than expected); under 35 phone app is close behind

Problem 16 A 3 by 3 Test

Row totals are 70, 90, 70; column totals are 70, 68, 92; the table total is 230. Expected counts: 21.30, 20.70, 28.00, 27.39, 26.61, 36.00, 21.30, 20.70, 28.00, all at least 5. $\chi^2 = 19.157$, df = 4,

p-value = 0.0007. Since $0.0007 < 0.05$, there is convincing evidence of an association between screen time and sleep quality.

$\chi^2 \approx 19.16$, df = 4, p-value ≈ 0.0007 ; reject H_0

Problem 17 Two Proportions and Chi-Square

(a) Pooled proportion = $117/290 = 0.4034$; $z = (0.4533 - 0.3500)/\sqrt{(0.4034(0.5966)(1/150 + 1/140))} = 1.792$. (b) The table is [68, 82; 49, 91], with expected counts 60.52, 89.48, 56.48, 83.52; $\chi^2 = 3.213$, p-value = 0.0731. (c) $z^2 = 3.213 = \chi^2$; the two-sided z test and the chi-square test give the same p-value.

$z \approx 1.792$; $\chi^2 \approx 3.213 = z^2$

Problem 18 Combining Categories

The column totals are 7, 20, 64 and 109, so the expected count for Never is $100(7)/200 = 3.5$ in both groups, which is below 5. Combine Never and Rarely: the new table is [11, 30, 59; 16, 34, 50], with all expected counts at least 5 (smallest 13.5). $\chi^2 = 1.919$, df = 2, p-value = 0.3831. Since $0.3831 > 0.05$, fail to reject H_0 : not convincing evidence that the distributions differ.

$\chi^2 \approx 1.92$, df = 2, p-value ≈ 0.383 ; fail to reject H_0

Problem 19 Association and Causation

(a) There is convincing evidence of an association between walking daily and reporting good sleep among adults. (b) No. This is an observational study; walkers may differ in other ways (for example, stress level or work schedule), so a cause-and-effect conclusion is not justified without random assignment.

Association yes; causation no, because there was no random assignment

Problem 20 An Experiment with Three Outcomes

H_0 : the distribution of grade outcomes is the same for all three methods; H_a : it is not. Random assignment is used and the expected counts (30.00, 32.67, 17.33, 30.00, 32.67, 17.33, 30.00, 32.67, 17.33) are all at least 5. $\chi^2 = 7.328$, df = 4, p-value = 0.1195. Since $0.1195 > 0.05$, we fail to reject H_0 : there is not convincing evidence that the grade distributions differ. Because this was a randomized experiment, a causal conclusion would have been allowed had the result been significant, but here there is no evidence of a difference.

$\chi^2 \approx 7.33$, df = 4, p-value ≈ 0.120 ; fail to reject H_0