

Inference for Means with t Procedures

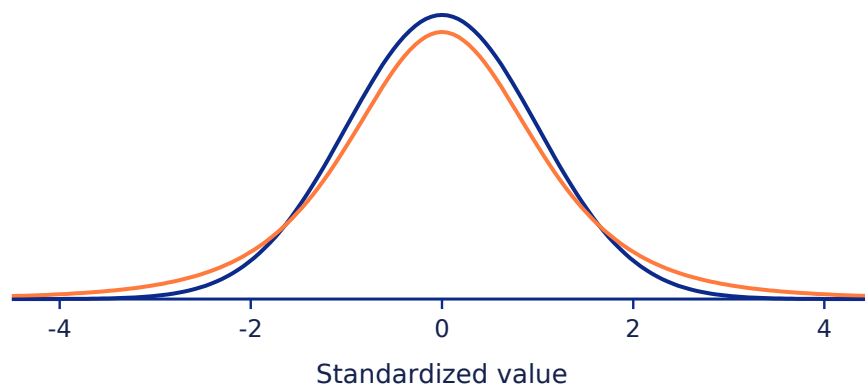
Grade 12 – AP Statistics math · Lesson · Standards: AP Statistics Unit 4 (2026-27)

Quick answer: Inference for means uses the t distribution, with $n - 1$ degrees of freedom, because σ is estimated by s . A t interval is $\bar{x} \pm t^* \cdot s/\sqrt{n}$; a t test uses $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$. Paired data use the differences, and two independent samples use a standard error that combines both variances.

Standards: AP Statistics Unit 4 (2026-27) · **Level:** Grade 12 AP Statistics

Why do we use a t distribution?

When σ is unknown we estimate it with the sample standard deviation s . The standardized statistic $t = (\bar{x} - \mu)/(s/\sqrt{n})$ follows a **t distribution** with $n - 1$ degrees of freedom: symmetric and bell-shaped like the normal curve, but with heavier tails that give wider intervals for small samples.



Conditions for t procedures

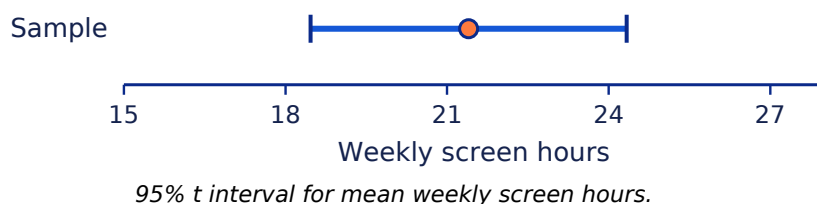
- **Random:** random sample or randomized experiment.
- **10%:** the sample is less than 10% of the population.
- **Normal/large sample:** the population is approximately normal, or $n \geq 30$, or a graph of the sample shows no strong skew or outliers.

How do you build a confidence interval for a mean?

One-sample t interval

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

Use t^* from the t distribution with $n - 1$ degrees of freedom (calculator: invT).

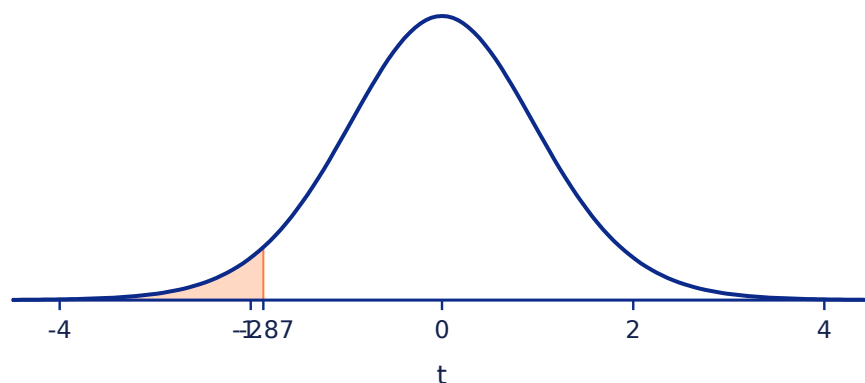


A random sample of 18 students reports $\bar{x} = 21.4$ hours of weekly screen time with $s = 5.9$. With $df = 17$, $t^* = 2.110$, so the standard error is 1.391 and the margin of error is 2.93. The interval is (18.47, 24.33) hours. We are 95% confident that the true mean weekly screen time of these students is in that interval.

How does a t test work?

Method: one-sample t test

1. State $H_0: \mu = \mu_0$, H_a , and the parameter in context.
2. Check random, 10% and normal/large sample conditions.
3. Compute $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$, $df = n - 1$, and the p-value (calculator: tcdf).
4. Compare the p-value with α and conclude in context.



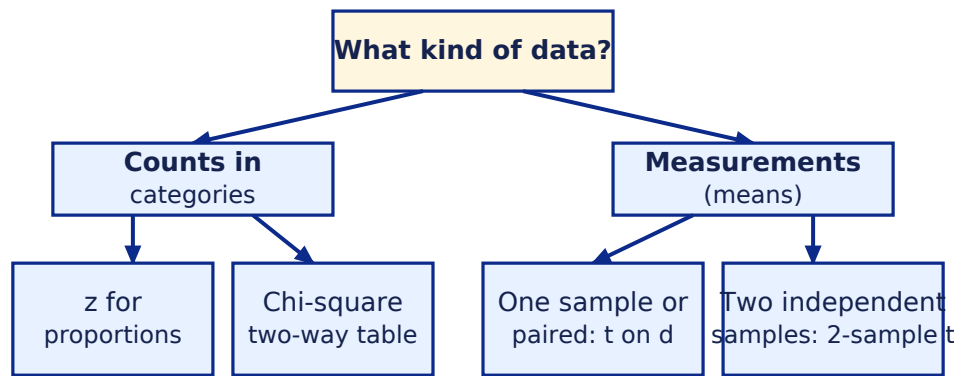
Worked example

Test $H_0: \mu = 24$ against $H_a: \mu < 24$. $t = (21.4 - 24)/1.391 = -1.870$, $df = 17$, p-value = 0.0394. Since $0.0394 < 0.05$, there is convincing evidence that the mean weekly screen time is below 24 hours.

What about paired data and two samples?

Paired data come from two measurements on the same individual or from matched pairs: compute the differences and run a one-sample t procedure on them. For **two independent samples**, the interval is $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$, and the test statistic is the difference divided by the same standard error; technology supplies the degrees of freedom. If 0 is outside the

interval, the data support a difference between the means.



Choosing an inference procedure.

Mathbot's tip

Ask first whether the two measurements are linked. The same person measured twice, or matched pairs, means paired: analyze the differences. Two unrelated groups mean two-sample.

Key points to remember

- t statistic: $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$, $df = n - 1$.
- Interval: $\bar{x} \pm t^* \cdot s/\sqrt{n}$.
- Paired data: analyze the differences with a one-sample t procedure.
- Two samples: $SE = \sqrt{(s_1^2/n_1 + s_2^2/n_2)}$.
- Check random, 10% and normal/large sample before using t.

Inference for Means with t Procedures: frequently asked questions

When do I use t instead of z for a mean?

Use t whenever the population standard deviation is unknown and estimated by s, which is almost always the case. The t distribution accounts for the extra uncertainty from estimating σ , especially when the sample is small.

How do I check the normality condition?

If $n \geq 30$ the Central Limit Theorem helps. For smaller samples, look at a dotplot, histogram or boxplot: it should be roughly symmetric with no strong skew and no outliers. Strong skew or outliers with a small sample make t procedures unreliable.

How do I tell paired data from two independent samples?

Paired data link each value in one set with a specific value in the other, such as before and after for the same person. Analyze the differences with a one-sample t procedure. If the groups are unrelated, use a two-sample t procedure.

What are the degrees of freedom for a one-sample t procedure?

$df = n - 1$. For paired data n is the number of pairs. For two independent samples, technology computes a non-integer df , and a safe conservative choice is the smaller of $n_1 - 1$ and $n_2 - 1$.

How can a confidence interval justify a claim about a mean?

If the claimed value is outside a 95% interval, there is convincing evidence against the claim at the 5% level for a two-sided test. If the value is inside, the data are consistent with the claim.