

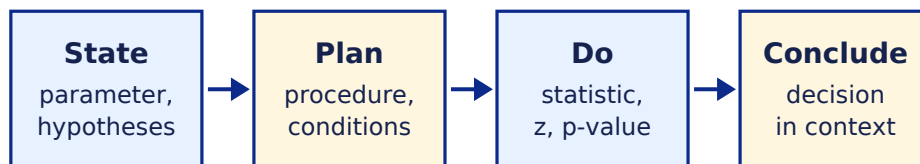
Inference for Proportions: Intervals and Tests

Grade 12 – AP Statistics math · Lesson · Standards: AP Statistics Unit 3 (2026-27)

Quick answer: When a categorical result is summarized by $\hat{p}$, two tools answer questions about the true share p . An interval of plausible values is $\hat{p} \pm z^* \cdot SE$. A significance test checks a claimed value p_0 with a z statistic and a p -value, while Type I errors, Type II errors and power measure the risk of a wrong decision.

Standards: AP Statistics Unit 3 (2026-27) · **Level:** Grade 12 AP Statistics

What are the steps of inference for a proportion?



The four steps of an inference procedure.

Conditions

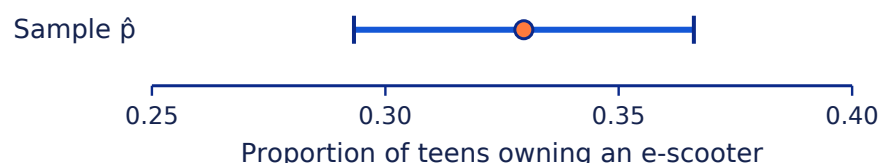
- **Random:** the data were produced by random sampling or a randomized experiment.
- **10%:** the sample is less than 10% of the population when sampling without replacement.
- **Large counts:** for an interval, at least 10 successes and 10 failures in the sample; for a test, at least 10 expected successes and 10 expected failures under H_0 .

How do you build and read a confidence interval?

One-proportion z interval

$$\hat{p} \pm z^* \sqrt{(\hat{p}(1 - \hat{p}))/n}$$

Critical values: $z^* = 1.645$ (90%), 1.960 (95%), 2.576 (99%).



95% confidence interval for the proportion of teens with an e-scooter.

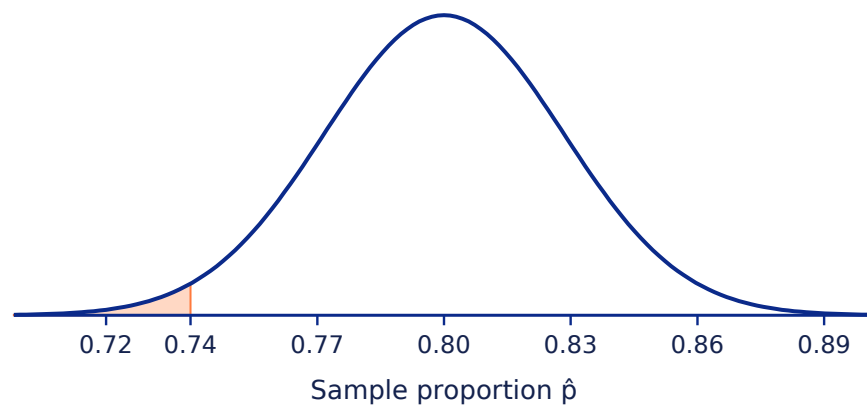
In a random sample of 640 teens, 211 own an e-scooter. Then $\hat{p} = 0.3297$, the standard error is 0.01858, and the margin of error is $1.96(0.01858) = 0.0364$. The interval is (0.293, 0.366). We are 95% confident that the true proportion of teens who own an e-scooter is in this interval. The 95% describes the method: about 95% of intervals made this way capture the parameter.

The margin of error is $z^* \times SE$, so the sample size needed for a margin of error ME is $n = (z^*/ME)^2 \hat{p}(1 - \hat{p})$, using $\hat{p} = 0.5$ if no estimate is available. A difference of two proportions uses $\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{(\hat{p}_1(1 - \hat{p}_1)/n_1 + \hat{p}_2(1 - \hat{p}_2)/n_2)}$; if 0 is outside the interval, the data support a difference.

How does a significance test work?

Method: one-proportion z test

1. State $H_0: p = p_0$ and H_a ($<$, $>$ or $\neq$), and a significance level α .
2. Verify random sampling, the 10% rule and large counts, with p_0 in the counts.
3. Calculate $z = (\hat{p} - p_0)/\sqrt{(p_0(1 - p_0)/n)}$ and the p-value.
4. If the p-value $\leq \alpha$, reject H_0 ; otherwise fail to reject, and conclude in context.



Null distribution of $\hat{p}$ ($n = 200$, $p_0 = 0.80$); the shaded tail is the p-value.

Worked example

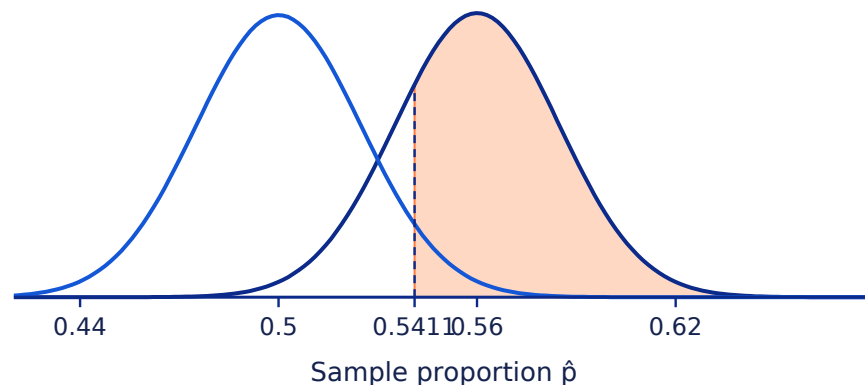
An airline claims 80% of flights are on time. In 200 random flights, 148 (74%) were on time. For $H_a: p < 0.80$, $z = (0.74 - 0.80)/0.0283 = -2.12$ and the p-value is 0.0169. Since $0.0169 < 0.05$, there is convincing evidence that fewer than 80% of flights are on time.

The **p-value** tells how likely it is, when H_0 is true, to see a sample result as far from p_0 as the one observed, or farther. It is not the probability that H_0 is true.

What are Type I and Type II errors, and what is power?

	H0 is true	H0 is false
Reject H0	Type I error (probability α)	Correct decision (power = $1 - \beta$)
Fail to reject H0	Correct decision	Type II error (probability β)

Power is the probability of rejecting H_0 when a specific alternative is true. It increases with a larger sample, a larger α , or a true value farther from p_0 . For $H_0: p = 0.50$, $n = 400$, $\alpha = 0.05$ and true $p = 0.56$, the power is 0.777.



Null (center 0.50) and alternative (center 0.56) curves; the shaded area right of 0.541 is the power.

Mathbot's tip

Never write that you accept H_0 . A large p-value only means the data do not provide convincing evidence against it.

Key points to remember

- Check random, 10% and large counts before using z procedures.
- Interval: $\hat{p} \pm z^* \sqrt{\hat{p}(1 - \hat{p})/n}$; critical values 1.645, 1.960, 2.576.
- Sample size for margin of error ME: $n = (z^*/ME)^2 \hat{p}(1 - \hat{p})$, rounded up.
- p-value: probability of a result at least as extreme when H_0 is true.
- Type I error probability is α ; power = $1 - \beta$.

Inference for Proportions: Intervals and Tests: frequently asked questions

What does a 95% confidence level mean?

If we repeated the sampling many times and built an interval each time with the same method, about 95% of the intervals would capture the true parameter. It does not mean a 95% chance that one fixed interval contains the parameter.

How do I choose the sample size for a given margin of error?

Solve $ME = z^* \sqrt{\hat{p}(1 - \hat{p})/n}$ for n , giving $n = (z^*/ME)^2 \hat{p}(1 - \hat{p})$. Without a prior estimate use $\hat{p} = 0.5$, which gives the largest value, and always round n up to the next whole number.

What is the p-value?

It is the probability, computed assuming H_0 is true, of getting a sample statistic at least as extreme

as the one observed in the direction of H_a . A small p-value is evidence against H_0 , but it is not the probability that H_0 is true.

How are Type I and Type II errors different?

A Type I error rejects a true H_0 , with probability α . A Type II error fails to reject a false H_0 , with probability β . Lowering α makes Type I errors rarer but makes Type II errors more likely unless the sample size increases.

Can a confidence interval be used for a two-sided test?

Yes. At the significance level $\alpha = 1 - \text{confidence level}$, if the hypothesized value is outside the interval, reject H_0 for a two-sided test. If it is inside, there is not convincing evidence against H_0 .