

Inference for Proportions: Intervals and Tests

Grade 12 – AP Statistics math · Worksheet Answer Key · Standards: AP Statistics Unit 3 (2026-27)

The answer key names each procedure, checks the conditions, shows the standard error and test statistic, and ends with a conclusion in context. Answers use the critical values 1.645, 1.960 and 2.576.

Standards: AP Statistics Unit 3 (2026-27) · **Level:** Grade 12 AP Statistics

Problem 1 Conditions for an Interval

Random: the sample was selected randomly. 10%: 180 is less than 10% of 4,200 (which is 420). Large counts: $n\hat{p} = 54 \geq 10$ and $n(1 - \hat{p}) = 126 \geq 10$. All conditions are met.

All three conditions are satisfied

Problem 2 A 90% Confidence Interval

$\hat{p} = 134/300 = 0.4467$. $SE = \sqrt{(0.4467 \times 0.5533/300)} = 0.02870$. $ME = 1.645 \times 0.02870 = 0.0472$. Interval: 0.4467 ± 0.0472 .

(0.399, 0.494)

Problem 3 Interpreting an Interval

We are 95% confident that the true proportion of all museum visitors who prefer the planetarium show is between 0.390 and 0.503. The confidence level means that if many random samples were taken and an interval built from each, about 95% of those intervals would capture the true proportion.

95% confident the true proportion is in (0.390, 0.503)

Problem 4 Margin of Error

$SE = \sqrt{(0.62 \times 0.38/500)} = 0.02171$. $ME = 2.576 \times 0.02171 = 0.0559$.

ME $\approx$ 0.0559

Problem 5 Using Intervals to Justify a Claim

A majority is supported only if every plausible value is above 0.50. School A: the interval (0.42, 0.56) contains 0.50, so not convincing. School B: (0.53, 0.67) is entirely above 0.50, so convincing evidence of a majority. School C: (0.38, 0.46) is entirely below 0.50, so the evidence points to a minority.

A: no; B: yes; C: no (evidence of fewer than half)

Problem 6 Sample Size for a Margin of Error

(a) $n = (1.96/0.03)^2(0.5)(0.5) = 1067.1$, round up to 1068. (b) $n = (1.96/0.03)^2(0.3)(0.7) = 896.4$, round up to 897.

(a) 1068; (b) 897

Problem 7 Writing Hypotheses

(a) $H_0: p = 0.80$, $H_a: p < 0.80$. (b) $H_0: p = 0.25$, $H_a: p > 0.25$. (c) $H_0: p = 0.50$, $H_a: p \neq 0.50$, where p is the true proportion of the relevant outcome.

(a) $p < 0.80$; (b) $p > 0.25$; (c) $p \neq 0.50$

Problem 8 One-Sided z Test

$\hat{p} = 148/200 = 0.74$. $SE = \sqrt{(0.8 \times 0.2/200)} = 0.0283$. $z = (0.74 - 0.80)/0.0283 = -2.12$. p-value = $P(Z < -2.12) = 0.0169$.

$z \approx -2.12$, p-value ≈ 0.0169

Problem 9 Two-Sided z Test

$\hat{p} = 0.56$, $SE = \sqrt{(0.5 \times 0.5/250)} = 0.0316$, $z = (0.56 - 0.50)/0.0316 = 1.897$. Two-sided p-value = $2(1 - 0.9711) = 0.0578$. Since $0.0578 > 0.05$, we fail to reject H_0 : there is not convincing evidence that the proportion differs from 0.50.

$z \approx 1.90$, p-value ≈ 0.0578 ; fail to reject H_0

Problem 10 Interpreting a p-Value

If exactly 50% of customers preferred the new design, the probability of getting a sample proportion at least as far from 0.50 as 0.56 (in either direction) in a random sample of 250 customers is about 0.058.

About 0.058 of samples would be this extreme if $p = 0.50$

Problem 11 Large Counts for a Test

$np_0 = 60(0.90) = 54 \geq 10$, but $n(1 - p_0) = 60(0.10) = 6 < 10$. The condition fails, so the z test is not appropriate for this sample size.

Condition not met: $n(1 - p_0) = 6 < 10$

Problem 12 Decisions and Significance Levels

0.032: reject at 0.05 ($0.032 \leq 0.05$), fail to reject at 0.01. 0.067: fail to reject at both levels.
0.0004: reject at both levels.

0.032: reject at 0.05 only; 0.067: fail to reject at both; 0.0004: reject at both

Problem 13 Type I and Type II Errors

Type I: concluding that the infection rate is greater than 4% when it is actually 4% (or less); consequence: unnecessary changes to procedures. Type II: failing to find evidence that the rate is above 4% when it truly is higher; consequence: patients continue to face a higher risk.

Type I: reject a true H_0 ; Type II: fail to reject a false H_0

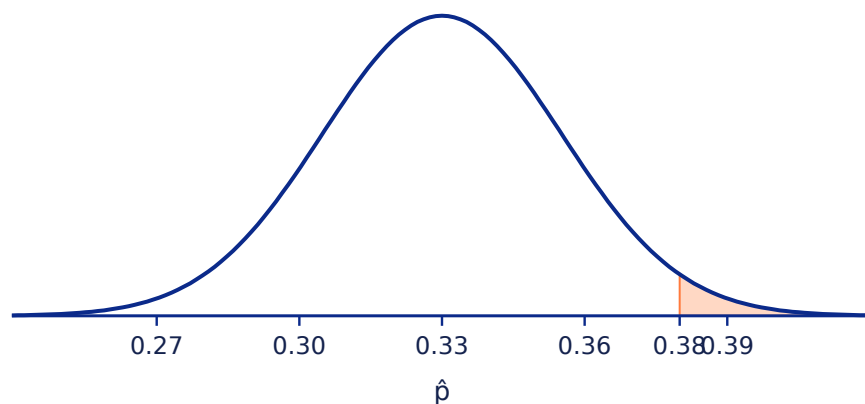
Problem 14 Factors That Affect Power

(a) A larger sample reduces the standard error, so the power increases. (b) A smaller α makes rejection harder, so power decreases. (c) A true value farther from p_0 is easier to detect, so power increases.

(a) increase; (b) decrease; (c) increase

Problem 15 A Complete One-Proportion Test

State: $H_0: p = 0.33$, $H_a: p > 0.33$, where p is the true proportion who recycle. **Plan:** one-proportion z test; random sample, 350 is less than 10% of the population, and $350(0.33) = 115.5$ and $350(0.67) = 234.5$ are at least 10. **Do:** $\hat{p} = 0.38$, $SE = 0.02513$, $z = 1.989$, p-value = 0.0233. **Conclude:** since the p-value is below 0.05, reject H_0 ; there is convincing evidence that more than 33% of residents recycle.



The p-value is the shaded upper tail.

$z \approx 1.99$, p-value ≈ 0.0233 ; reject H_0

Problem 16 Interval for a Difference

Conditions: both samples are random and independent; all counts 96, 54, 78, 82 are at least 10.
 $\hat{p}_1 = 0.6400$, $\hat{p}_2 = 0.4875$, difference = 0.1525. $SE = \sqrt{(0.6400(0.3600)/150 + 0.4875(0.5125)/160)} = 0.0557$. $ME = 1.96 \times 0.0557 = 0.1091$.

(0.043, 0.262)

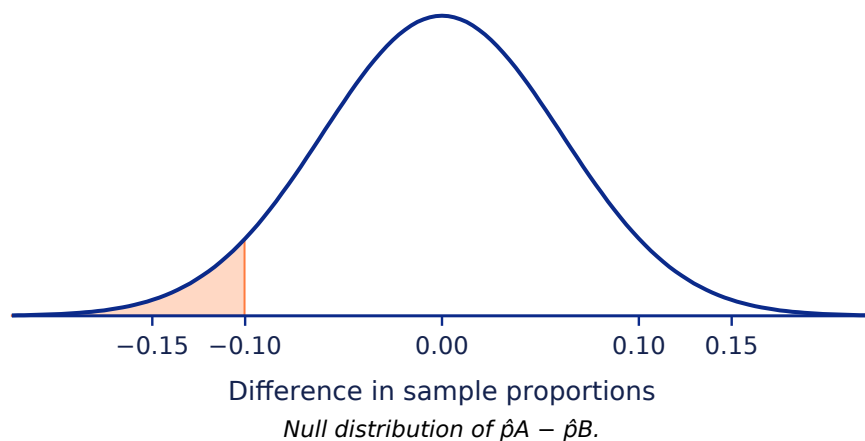
Problem 17 Using the Interval to Decide

The interval contains only positive values and does not include 0, so there is convincing evidence that the pass rate is higher with the program. We are 95% confident that the true pass rate with the program is between 4.3 and 26.2 percentage points higher than without it.

Yes; 0 is not in the interval

Problem 18 Two-Proportion z Test

$H_0: p_A = p_B$; $H_a: p_A \neq p_B$. Pooled proportion = $75/230 = 0.3261$. Conditions: random independent samples and all expected counts are at least 10. $SE = \sqrt{(0.3261(0.6739)(1/120 + 1/110))} = 0.0619$. $z = (0.3750 - 0.2727)/0.0619 = 1.653$; two-sided p-value = 0.0984. Since $0.0984 > 0.05$, fail to reject: not convincing evidence that the proportions differ.



$z \approx 1.65$, p-value ≈ 0.098 ; fail to reject H_0

Problem 19 Computing Power

Reject H_0 when $\hat{p}$ exceeds $0.40 + 1.645\sqrt{(0.4 \times 0.6/300)} = 0.40 + 1.645(0.02828) = 0.4465$. If $p = 0.47$, the standard deviation of $\hat{p}$ is $\sqrt{(0.47 \times 0.53/300)} = 0.02882$. Power = $P(\hat{p} > 0.4465) = P(Z > -0.815) = 0.7923$.

Power ≈ 0.792

Problem 20 Poll Planning and Claim

(a) $SE = \sqrt{(0.54 \times 0.46/900)} = 0.01661$; $ME = 1.96 \times 0.01661 = 0.0326$; interval (0.507, 0.573).
 (b) The interval is entirely above 0.50, so yes. (c) ME is proportional to $1/\sqrt{n}$; halving it requires 4 times the sample size, or 3,600 voters.

(a) (0.507, 0.573); (b) yes; (c) 4 times, 3,600 voters