

Probability Rules and Random Variables

Grade 12 – AP Statistics math · Lesson · Standards: AP Statistics Unit 2 (2026-27)

Quick answer: Probability describes chance with numbers from 0 to 1. Two-way tables give joint, marginal and conditional relative frequencies; the complement, addition and multiplication rules combine events; and a discrete random variable has a mean and standard deviation computed from its probability distribution.

Standards: AP Statistics Unit 2 (2026-27) · **Level:** Grade 12 AP Statistics

How do you read a two-way table?

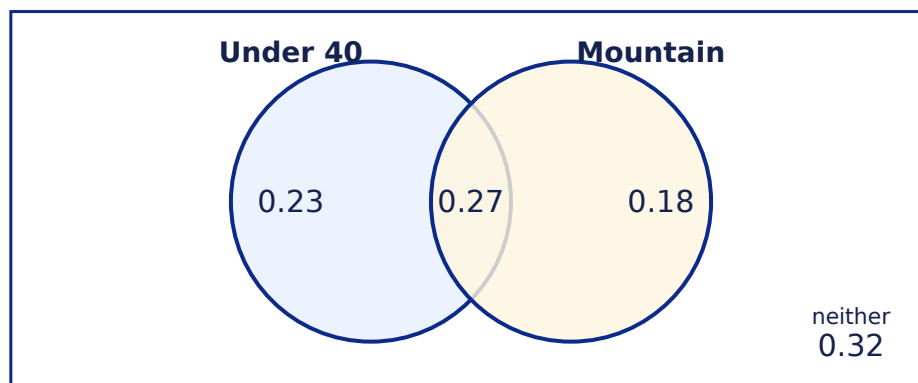
A hiking club of 200 members was classified by age group and favorite trail type.

	Mountain	Coastal	Total
Under 40	54	46	100
40 and over	36	64	100
Total	90	110	200

Three kinds of relative frequency

- **Joint:** a cell over the grand total, such as $54/200 = 0.27$ for under 40 and mountain.
- **Marginal:** a total over the grand total, such as $90/200 = 0.45$ for mountain.
- **Conditional:** a cell over a row or column total, such as $54/100 = 0.54$ for mountain among members under 40.

Since 0.54 differs from 0.36 (the same proportion among members 40 and over), age group and trail preference are associated.



Probabilities for age group and favorite trail.

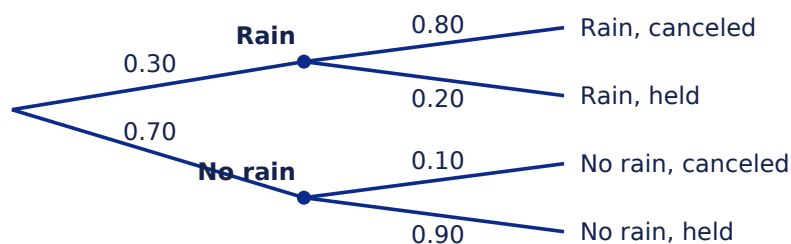
Which probability rules should you know?

Rules

- Complement: $P(\text{not } A) = 1 - P(A)$.
- Addition: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- Mutually exclusive events cannot occur together: $P(A \text{ and } B) = 0$.
- Conditional: $P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$
- Independent events satisfy $P(A | B) = P(A)$, equivalently $P(A \text{ and } B) = P(A) \cdot P(B)$.

In the Venn diagram, $P(\text{under 40 or mountain}) = 0.23 + 0.27 + 0.18 = 0.68$. Mutually exclusive is not the same as independent: exclusive events with positive probabilities are always dependent.

How does a tree diagram help?

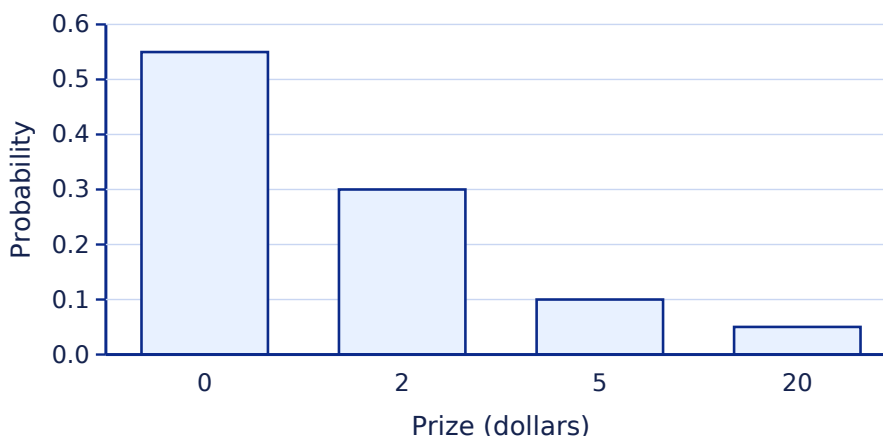


A two-stage tree for the weather and the hike.

Worked example

$P(\text{canceled}) = 0.30(0.80) + 0.70(0.10) = 0.24 + 0.07 = 0.31$. Given that the hike was canceled, $P(\text{rain} | \text{canceled}) = 0.24/0.31 \approx 0.774$.

How do you describe a discrete random variable?



Distribution of the prize at a fair game.

Mean and standard deviation of X

$$\mu_x = \sum x_i p_i \quad \sigma_x = \sqrt{(\sum (x_i - \mu_x)^2 p_i)}$$

For the prize game, $\mu = 0(0.55) + 2(0.30) + 5(0.10) + 20(0.05) = 2.10$ dollars and $\sigma \approx 4.39$ dollars. The mean is the long-run average prize, not a prize you can win in one play.

Method: simulation

1. State the random process and the event of interest.
2. Assign digits so each outcome has the right probability.
3. Run one trial, then record the result.
4. Repeat many trials; the proportion with the event estimates its probability.

Mathbot's tip

When a problem says given that, the denominator shrinks to the given group. Circle the group first, then count.

Key points to remember

- Conditional probability: $P(A | B) = P(A \text{ and } B)/P(B)$.
- Independent events: $P(A \text{ and } B) = P(A) \times P(B)$.
- Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- Mean of a discrete random variable: $\mu = \sum x \cdot p$.
- A simulation estimates a probability by repeating a random process many times.

Probability Rules and Random Variables: frequently asked questions

What is the difference between mutually exclusive and independent events?

Mutually exclusive events cannot happen together, so $P(A \text{ and } B) = 0$. Independent events do not affect each other's probability, so $P(A \text{ and } B) = P(A)P(B)$. Two events with positive probabilities cannot be both.

How do I compute a conditional probability from a two-way table?

Restrict attention to the given row or column, then divide the count in the cell of interest by that row or column total. $P(\text{mountain} | \text{under } 40)$ uses the under-40 row total as the denominator.

What does the mean of a random variable represent?

It is the long-run average value of the variable over many repetitions of the random process. It need not be a possible value, and it describes the average, not any single outcome.

When do I use the addition rule without subtracting?

Only for mutually exclusive events. Otherwise subtract $P(A \text{ and } B)$ to avoid counting the overlap twice. A Venn diagram or two-way table shows the overlap clearly.

How do I design a simulation?

Assign digits so the probabilities match, such as 0-2 for a 30% event, define one trial, run many trials and count how often the event occurs. The proportion estimates the probability, and more trials give a more stable estimate.