

Sampling Distributions of Proportions and Means

Grade 12 – AP Statistics math · Lesson · Standards: AP Statistics Unit 2 (2026-27)

Quick answer: A sampling distribution describes how a statistic such as $\bar{x}$ or $\hat{p}$ varies from sample to sample. Its center shows bias, its spread shows variability, and its shape is approximately normal under the Central Limit Theorem or the conditions $np \geq 10$ and $n(1 - p) \geq 10$.

Standards: AP Statistics Unit 2 (2026-27) · **Level:** Grade 12 AP Statistics

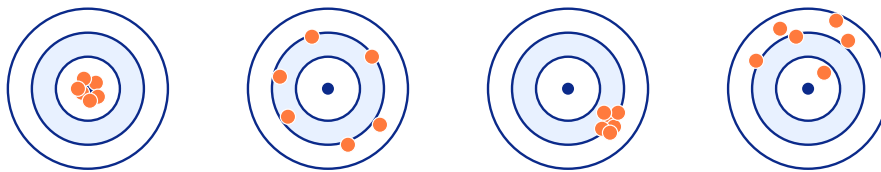
What is a sampling distribution?

A **parameter** describes a population (μ, p); a **statistic** describes a sample ($\bar{x}, \hat{p}$). A **sampling distribution** shows every value a statistic takes over all possible samples of the same size.

Bias and variability

A statistic is **unbiased** if the center of its sampling distribution equals the parameter.

Variability is the spread of that distribution; it shrinks as the sample size grows.



low bias, low spread low bias, high spread high bias, low spread high bias, high spread

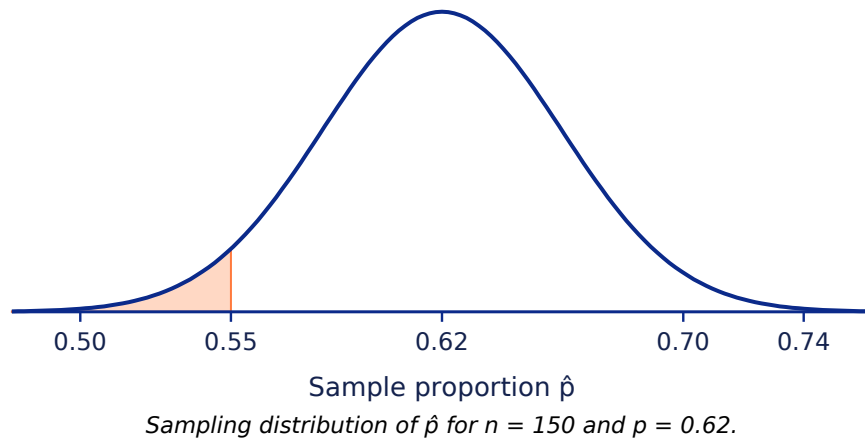
The bullseye is the true parameter; dots are sample statistics.

What is the sampling distribution of a sample proportion?

Sample proportion

$$\mu_{\hat{p}} = p \quad \sigma_{\hat{p}} = \sqrt{(p(1-p))/n}$$

Conditions: random sample, n less than 10% of the population, $np \geq 10$ and $n(1 - p) \geq 10$. Then the shape is approximately normal.



Worked example

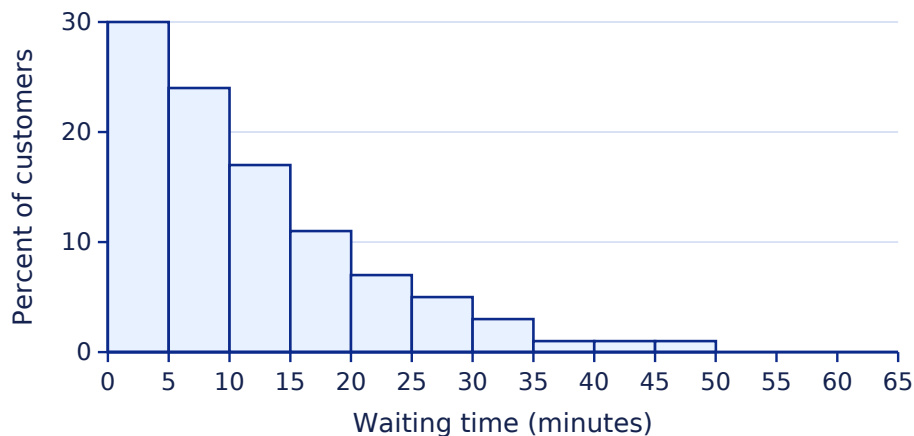
If 62% of voters support a measure and $n = 150$, then $\sigma = \sqrt{(0.62 \times 0.38/150)} = 0.0396$. $P(\hat{p} < 0.55)$: $z = (0.55 - 0.62)/0.0396 = -1.77$, so the probability is 0.0387.

What is the sampling distribution of a sample mean?

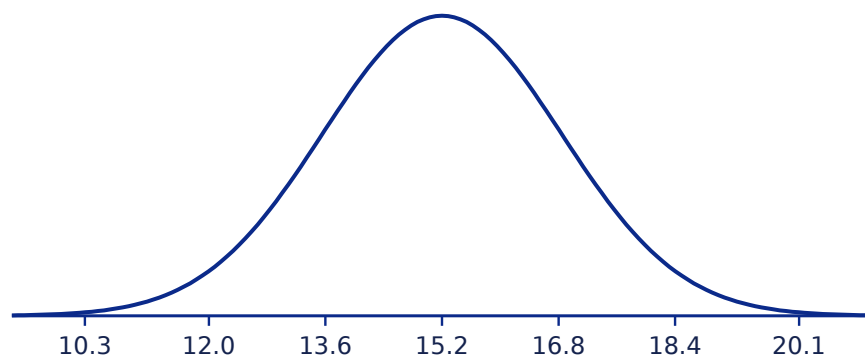
Sample mean

$$\mu_{\bar{x}} = \mu \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

If the population is normal, $\bar{x}$ is normal for any n . Otherwise the **Central Limit Theorem** says $\bar{x}$ is approximately normal when n is large (at least 30 is a common guide).



A skewed right population of waiting times.



Sampling distribution of $\bar{x}$ for $n = 45$: nearly normal.

Waiting times have mean 15.2 and standard deviation 11 minutes, with a skewed population. For samples of 45, the center is 15.2 and $\sigma = 11/\sqrt{45} = 1.64$, so the sampling distribution is nearly normal even though the population is not.

What about differences of two statistics?

Statistic	Mean	Standard deviation
$\hat{p}_1 - \hat{p}_2$	$p_1 - p_2$	$\sqrt{(p_1(1 - p_1)/n_1 + p_2(1 - p_2)/n_2)}$
$\bar{x}_1 - \bar{x}_2$	$\mu_1 - \mu_2$	$\sqrt{(\sigma_1^2/n_1 + \sigma_2^2/n_2)}$

Variances add for independent samples, so the spread of a difference is larger than the spread of either statistic alone.

Mathbot's tip

Quadrupling the sample size cuts the standard deviation of the statistic in half, not to a quarter. Bigger samples reduce variability but never fix bias.

Key points to remember

- $\hat{p}$ has mean p and standard deviation $\sqrt{(p(1 - p)/n)}$.
- $\bar{x}$ has mean μ and standard deviation $\sigma/\sqrt{n}$.
- The Central Limit Theorem makes $\bar{x}$ approximately normal for large n .
- Larger samples reduce variability but do not remove bias.
- For differences of independent statistics, subtract the means and add the variances.

Sampling Distributions of Proportions and Means: frequently asked questions

What is the difference between a population distribution and a sampling distribution?

A population distribution shows the values of individuals. A sampling distribution shows the values

of a statistic, such as $\bar{x}$, over all possible samples of the same size. The sampling distribution has the same center as the population for $\bar{x}$ but much less spread.

What does the Central Limit Theorem say?

For a large random sample, the sampling distribution of $\bar{x}$ is approximately normal, whatever the shape of the population, with mean μ and standard deviation $\sigma/\sqrt{n}$. Strongly skewed populations need larger samples than symmetric ones.

When can I use the formula $\sigma = \sqrt{(p(1 - p)/n)}$ for the standard deviation of $\hat{p}$?

When the sample is random and less than 10% of the population, so that observations are nearly independent. The normal shape also requires $np \geq 10$ and $n(1 - p) \geq 10$.

How do bias and variability differ?

Bias is a systematic miss: the sampling distribution is centered away from the parameter. Variability is how widely the statistic scatters from sample to sample. Good sampling methods give low bias, and large samples give low variability.

Why does the standard deviation of a difference use addition?

For independent samples the variances add, whether you subtract or add the statistics. The standard deviation is the square root of the sum of the two variances, so it is larger than either standard deviation alone.