

Sampling Distributions of Proportions and Means

Grade 12 – AP Statistics math · Worksheet Answer Key · Standards: AP Statistics Unit 2 (2026-27)

The answer key checks conditions before using a normal model, shows the mean and standard deviation of every sampling distribution, and states the probability with an interpretation when asked.

Standards: AP Statistics Unit 2 (2026-27) · **Level:** Grade 12 AP Statistics

Problem 1 Parameter or Statistic

(a) It describes the whole population: a parameter, μ . (b) It describes a sample: a statistic, $\hat{p}$. (c) It describes a sample of batteries: a statistic, $\bar{x}$.

(a) parameter μ ; (b) statistic $\hat{p}$; (c) statistic $\bar{x}$

Problem 2 Center and Spread of $\hat{p}$

$\mu = p = 0.40$. $\sigma = \sqrt{(0.40 \times 0.60/100)} = \sqrt{0.0024} = 0.0490$.

mean 0.40, standard deviation $\approx$ 0.0490

Problem 3 Checking Conditions for $\hat{p}$

The sample is under 10% of 5,000 ($120 < 500$). For $n = 120$: $np = 9.6 < 10$, so the normal approximation is not appropriate. For $n = 250$: $np = 20 \geq 10$ and $n(1 - p) = 230 \geq 10$, so it is appropriate.

Not normal for $n = 120$ ($np = 9.6$); approximately normal for $n = 250$

Problem 4 Probability for a Sample Proportion

$\sigma = \sqrt{(0.35 \times 0.65/200)} = 0.0337$. Conditions: $np = 70$ and $n(1 - p) = 130$ are both at least 10. $z = (0.40 - 0.35)/0.0337 = 1.48$. $P(\hat{p} \geq 0.40) = 0.0691$.

$\approx$ 0.0691

Problem 5 Reading a Simulated Sampling Distribution

The distribution is roughly symmetric and mound-shaped. The mean of the 40 values is 0.283, close to 0.30, which suggests $\hat{p}$ is unbiased. The standard deviation of the simulated values is 0.059, near the theoretical $\sqrt{(0.30 \times 0.70/50)} = 0.065$.

Symmetric, center ≈ 0.28 , spread ≈ 0.059 (theory 0.065)

Problem 6 Sample Size and Spread

$n = 100$: $\sqrt{(0.5 \times 0.5/100)} = 0.050$. $n = 400$: $\sqrt{(0.5 \times 0.5/400)} = 0.025$. The standard deviation is cut in half.

0.050 versus 0.025: quadrupling n halves the standard deviation

Problem 7 Matching Bias and Variability

(a) A: centered on the bullseye with little scatter. (b) C: dots are tightly grouped but away from the target. (c) D. (d) Method B is unbiased but scattered; taking larger samples reduces variability.

(a) A; (b) C; (c) D; (d) increase the sample size

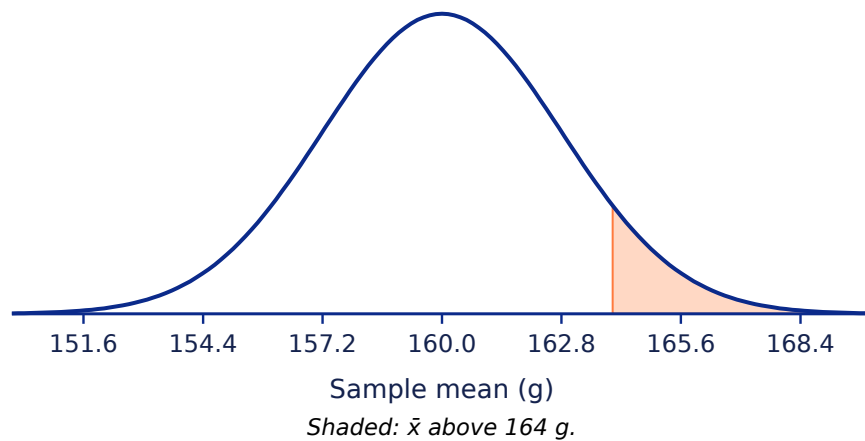
Problem 8 Mean and SD of a Sample Mean

$\mu_{\bar{x}} = 72$. $\sigma_{\bar{x}} = 9/\sqrt{36} = 9/6 = 1.5$.

mean 72, standard deviation 1.5

Problem 9 Normal Population, Sample Mean

The population is normal, so $\bar{x}$ is normal with mean 160 and $\sigma = 14/\sqrt{25} = 2.8$. $z = (164 - 160)/2.8 = 1.43$. $P(\bar{x} > 164) = 0.0766$.



≈ 0.0766

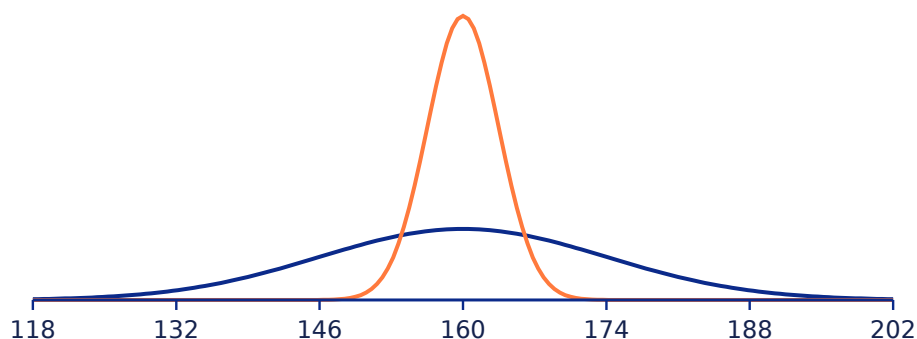
Problem 10 Central Limit Theorem Application

The population is skewed, but $n = 45 \geq 30$, so by the Central Limit Theorem $\bar{x}$ is approximately normal with mean 15.2 and $\sigma = 11/\sqrt{45} = 1.640$. $z = (18 - 15.2)/1.640 = 1.71$, and the probability is 0.0439.

≈ 0.0439

Problem 11 Individual Versus Mean

(a) $z = (150 - 160)/14 = -0.71$, probability 0.2375. (b) $\sigma_{\bar{x}} = 14/4 = 3.5$, $z = -2.86$, probability 0.0021. (c) Averages vary much less than individuals, so a mean far from 160 is rarer than an individual far from 160.



Individuals (wide) and sample means (narrow).

(a) 0.2375; (b) 0.0021

Problem 12 Sample Size for a Target Standard Deviation

Solve $14/\sqrt{n} \leq 2$: $\sqrt{n} \geq 7$, so $n \geq 49$.

$n = 49$

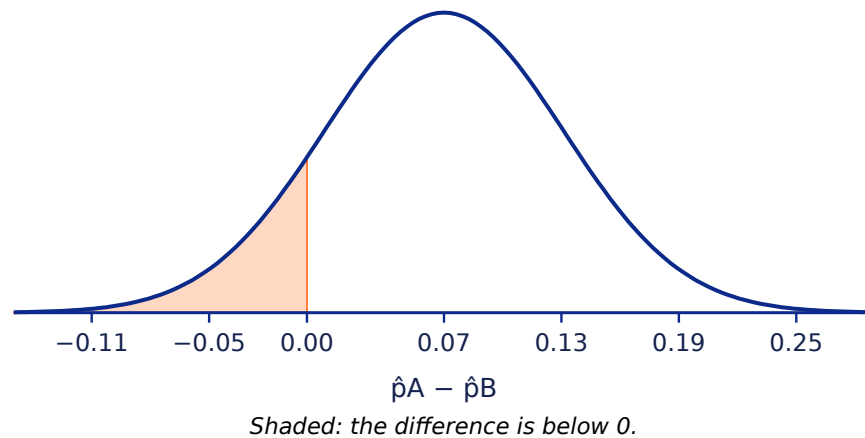
Problem 13 Difference of Two Proportions

Mean = $0.55 - 0.48 = 0.07$. $\sigma = \sqrt{(0.55 \times 0.45/120 + 0.48 \times 0.52/150)} = \sqrt{(0.002063 + 0.001664)} = 0.0610$.

mean 0.07, standard deviation ≈ 0.0610

Problem 14 Probability for a Difference of Proportions

Conditions: $120(0.55) = 66$, $120(0.45) = 54$, $150(0.48) = 72$, $150(0.52) = 78$, all at least 10. $z = (0 - 0.07)/0.0610 = -1.15$. $P(\text{difference} < 0) = 0.1258$.



≈ 0.1258

Problem 15 Difference of Two Sample Means

Mean = $82 - 76 = 6$. $\sigma = \sqrt{(12^2/40 + 10^2/35)} = \sqrt{(3.600 + 2.857)} = 2.541$.

mean 6, standard deviation ≈ 2.541

Problem 16 Probability for a Difference of Means

$z = (10 - 6)/2.541 = 1.57$. $P(\bar{x}_A - \bar{x}_B > 10) = 0.0577$.

≈ 0.0577

Problem 17 A Complete Sampling Distribution

The population mean is $(3 + 5 + 9 + 11)/4 = 7$.

$\bar{x}$	Pairs	Probability
3	1	1/16
4	2	2/16
5	1	1/16
6	2	2/16
7	4	4/16
8	2	2/16
9	1	1/16
10	2	2/16
11	1	1/16

The mean of the 16 values of $\bar{x}$ is 7, equal to μ , so $\bar{x}$ is an unbiased estimator of μ .

Mean of $\bar{x} = 7 = \mu$

Problem 18 Quality Control Proportion

Conditions: $500(0.04) = 20 \geq 10$, $500(0.96) = 480 \geq 10$, and the shipment is very large. $\sigma = \sqrt{(0.04 \times 0.96/500)} = 0.00876$. $z = (0.06 - 0.04)/0.00876 = 2.28$. $P(\hat{p} > 0.06) = 0.0112$.

≈ 0.0112

Problem 19 Is the Sample Mean Unusual?

$\sigma_{\bar{x}} = 6/\sqrt{36} = 1$. $z = (338 - 340)/1 = -2.0$. $P(\bar{x} \leq 338) = 0.0228$. A probability of about 2.3% is small, so a sample mean this low would be unusual if the mean really were 340, which is moderate evidence that the true mean is lower.

≈ 0.0228 ; moderate evidence against the claim

Problem 20 Comparing Two Clinics

Mean of $\hat{p}_A - \hat{p}_B = 0.08$. $\sigma = \sqrt{(0.70 \times 0.30/140 + 0.62 \times 0.38/120)} = \sqrt{(0.001500 + 0.001963)} = 0.0589$. $z = (0.15 - 0.08)/0.0589 = 1.19$. Probability = 0.1171. A gap this large happens about 12% of the time by chance alone, even though the true gap is 0.08.

≈ 0.1171