

Arc Length, Sector Area and Radian Measure

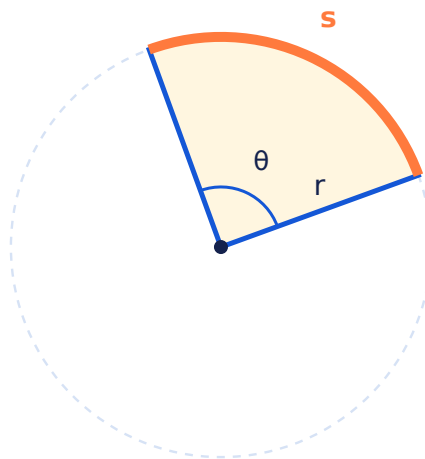
Grade 10 – Geometry math · Lesson · Standards: G-C.B.5

Quick answer: Arc length and sector area are proportional to the central angle. An arc is the fraction $\theta/360$ of the circumference $2\pi r$, and a sector is the same fraction of the area πr^2 . A radian is arc length divided by radius, so 180° equals π radians.

Standards: G-C.B.5 · **Level:** Grade 10 Geometry

How do you find the length of an arc?

An arc is a fraction of the circumference, and the fraction is the central angle over 360° . A sector is a fraction of the disk, with the same fraction.



Sector with radius r , central angle θ and arc length s .

Arc length and sector area (degrees)

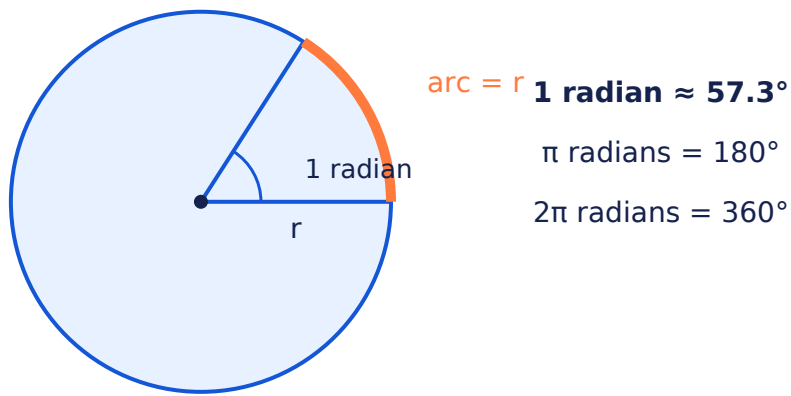
$$s = \frac{\theta}{360} \cdot 2\pi r \quad \text{Area} = \frac{\theta}{360} \cdot \pi r^2$$

The two answers scale together: doubling the angle doubles both the arc and the area of the sector.

What is a radian?

Radian measure

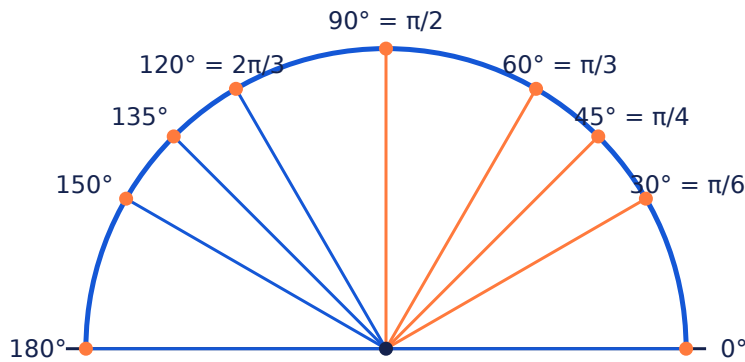
The radian measure of a central angle is the length of its arc divided by the radius: $\theta = s \div r$. A radian is the angle whose arc is exactly as long as the radius.



An arc as long as the radius marks 1 radian, about 57.3°.

How do you convert between degrees and radians?

A full turn is 360°, which is 2π radians, so $180^\circ = \pi$ radians. Multiply degrees by $\pi/180$, or radians by $180/\pi$.



Common angles in degrees and radians.

Formulas in radians

$$s = r\theta \quad \text{Area} = \frac{1}{2} r^2 \theta$$

Method

1. Decide whether the angle is in degrees or radians.
2. Write the fraction of the circle: $\theta/360$ or $\theta/(2\pi)$.
3. Multiply it by the circumference $2\pi r$ for an arc or by the area πr^2 for a sector.
4. Include units: linear units for arcs, square units for sectors.

Example

A sector has radius 10 inches and a central angle of 72°. Find the arc length and the area.

The fraction is $72/360 = \frac{1}{5}$. Arc length = $\frac{1}{5} \cdot 20\pi = 4\pi \approx 12.57$ in. Area = $\frac{1}{5} \cdot 100\pi = 20\pi \approx 62.83$ in².

Common mistake

Using the circumference formula for a sector area, or the area formula for an arc. Arc length uses $2\pi r$; sector area uses πr^2 .

Mathbot's tip Write the fraction of the circle first. If it simplifies to something friendly, like $1/5$ or $3/8$, the rest is easy arithmetic.

Key points to remember

- Arc length = $(\theta/360) \cdot 2\pi r$ and sector area = $(\theta/360) \cdot \pi r^2$ with θ in degrees.
- Radian measure $\theta = s \div r$; with radians, $s = r\theta$ and area = $\frac{1}{2}r^2\theta$.
- $180^\circ = \pi$ radians, so multiply degrees by $\pi/180$ to convert.
- Similar sectors with the same angle have arc lengths in the ratio of the radii.

Arc Length, Sector Area and Radian Measure: frequently asked questions

Why is arc length proportional to the central angle?

A circle has 360° around its center and one circumference around its edge. An angle that is a fraction of 360° cuts off the same fraction of the circumference.

What is one radian?

One radian is the central angle whose arc has the same length as the radius. Since the circumference is 2π radii long, a full circle measures 2π radians.

How do I convert 150° to radians?

Multiply by $\pi/180$: $150 \cdot \pi/180 = 5\pi/6$. The fraction $150/180$ reduces to $5/6$, so the angle is five sixths of π radians.

Why is the sector area $\frac{1}{2}r^2\theta$ in radians?

The sector is the fraction $\theta/(2\pi)$ of the disk. Multiplying by πr^2 leaves $\frac{1}{2}r^2\theta$ because the π cancels.

Do arc length and sector area use different units?

Yes. Arc length is a length in inches, feet or meters. Sector area is a two-dimensional measure in square inches, square feet or square meters.