

Arc Length, Sector Area and Radian Measure

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-C.B.5

Every solution writes the fraction of the circle first, then the formula, then the exact value and a rounded decimal. Compare the exact multiple of π before rounding to find where a calculation drifted.

Standards: G-C.B.5 · **Level:** Grade 10 Geometry

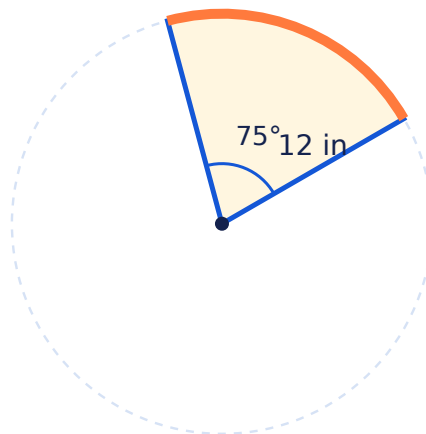
Problem 1 Arc Length in Degrees

$$s = \frac{40}{360} \cdot 2\pi(9) = \frac{1}{9} \cdot 18\pi = 2\pi.$$

$$2\pi \approx 6.28.$$

$$2\pi \text{ cm} \approx 6.28 \text{ cm}$$

Problem 2 Sector Area in Degrees



Fraction of the circle: $75/360 = 5/24$.

$$\text{Area} = \frac{75}{360} \cdot \pi(12)^2 = \frac{5}{24} \cdot 144\pi = 30\pi.$$

$$30\pi \approx 94.25.$$

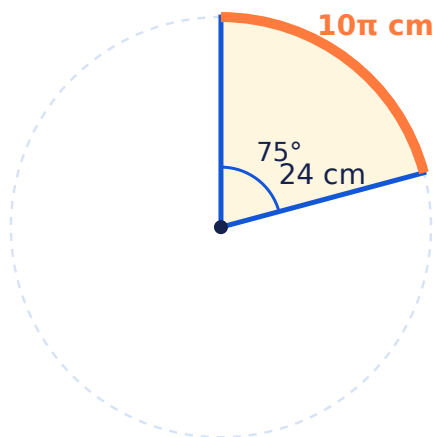
$$30\pi \text{ in}^2 \approx 94.25 \text{ in}^2$$

Problem 3 Arc Length of a Wide Angle

$$s = \frac{132}{360} \cdot 2\pi(15) = \frac{11}{30} \cdot 30\pi = 11\pi \approx 34.56.$$

$\approx 34.6 \text{ ft}$

Problem 4 Central Angle from an Arc



The angle is 75° .

Circumference = 48π . Fraction = $\frac{10\pi}{48\pi} = \frac{5}{24}$.

Angle = $\frac{5}{24} \cdot 360^\circ = 75^\circ$.

75°

Problem 5 Degrees to Radians

Multiply by $\frac{\pi}{180}$.

(a) $135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$. (b) $210 \cdot \frac{\pi}{180} = \frac{7\pi}{6}$. (c) $80 \cdot \frac{\pi}{180} = \frac{4\pi}{9}$.

(a) $3\pi/4$ (b) $7\pi/6$ (c) $4\pi/9$

Problem 6 Radians to Degrees

Multiply by $\frac{180}{\pi}$.

(a) $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150^\circ$. (b) $\frac{3\pi}{10} \cdot \frac{180}{\pi} = 54^\circ$. (c) $2 \cdot \frac{180}{\pi} \approx 114.59^\circ$.

(a) 150° (b) 54° (c) $\approx 114.6^\circ$

Problem 7 Radian Measure from Arc and Radius

$\theta = \frac{s}{r} = \frac{14}{8} = 1.75$ radians.

In degrees: $1.75 \cdot \frac{180}{\pi} \approx 100.27^\circ$.

$$1.75 \text{ rad} \approx 100.3^\circ$$

Problem 8 Arc Length with Radians

$$s = r\theta = 6.5 \cdot 2.4 = 15.6.$$

$$\mathbf{15.6 \text{ m}}$$

Problem 9 Sector Area with Radians

$$\text{Area} = \frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 100 \cdot 1.2 = 60.$$

$$\mathbf{60 \text{ ft}^2}$$

Problem 10 Radius from a Sector Area

$$\frac{120}{360} \cdot \pi r^2 = 54\pi, \text{ so } \frac{1}{3} r^2 = 54 \text{ and } r^2 = 162.$$

$$r = \sqrt{162} = 9\sqrt{2} \approx 12.73.$$

$$\mathbf{r = 9\sqrt{2} \approx 12.7 \text{ in}}$$

Problem 11 Radius from an Arc Length

$$\frac{84}{360} = \frac{7}{30}. \text{ So } \frac{7}{30} \cdot 2\pi r = 7\pi, \text{ giving } \frac{14\pi r}{30} = 7\pi.$$

$$r = 7 \cdot 30 \div 14 = 15.$$

$$\mathbf{r = 15 \text{ cm}}$$

Problem 12 Perimeter of a Sector

$$\text{Arc} = \frac{45}{360} \cdot 2\pi(8) = \frac{1}{8} \cdot 16\pi = 2\pi \approx 6.28.$$

$$\text{Perimeter} = 8 + 8 + 2\pi = 16 + 2\pi \approx 22.28.$$

$$\mathbf{\approx 22.28 \text{ m}}$$

Problem 13 Similar Sectors

$$\text{Arc lengths: } \frac{1}{6} \cdot 2\pi(5) = \frac{5\pi}{3} \text{ and } \frac{1}{6} \cdot 2\pi(15) = 5\pi. \text{ Ratio } 5\pi \div (\frac{5\pi}{3}) = 3.$$

$$\text{Areas: } \frac{1}{6} \cdot 25\pi = \frac{25\pi}{6} \text{ and } \frac{1}{6} \cdot 225\pi = \frac{75\pi}{2}. \text{ Ratio} = 9.$$

The radius ratio is 3; arcs scale by 3 and areas by $3^2 = 9$.

Arcs 3 : 1; areas 9 : 1

Problem 14 Area of a Circular Segment

Sector area = $\frac{60}{360} \cdot 100\pi \approx 52.36$.

Triangle area = $\frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 60^\circ \approx 43.3$.

Segment = $52.36 - 43.3 = 9.06$.

$\approx 9.06 \text{ cm}^2$

Problem 15 Pizza Slice

Radius = 9 in. Each slice has central angle $360^\circ \div 8 = 45^\circ$.

Area = $\frac{1}{8} \cdot 81\pi \approx 31.81$. Crust = $\frac{1}{8} \cdot 18\pi = 2.25\pi \approx 7.07$.

Area $\approx 31.81 \text{ in}^2$; crust $\approx 7.07 \text{ in}$

Problem 16 Windshield Wiper

(a) Tip path = $\frac{110}{360} \cdot 2\pi(24) = \frac{11}{36} \cdot 48\pi = \frac{44\pi}{3} \approx 46.08 \text{ in}$.

(b) The blade clears the sector of radius 24 minus the sector of radius 6: $\frac{11}{36} \cdot \pi(24^2 - 6^2) = \frac{11}{36} \cdot 540\pi = 165\pi \approx 518.36 \text{ in}^2$.

(a) $\approx 46.1 \text{ in}$ (b) $\approx 518.4 \text{ in}^2$

Problem 17 Lawn Sprinkler

Area = $\frac{220}{360} \cdot \pi(14^2) = \frac{11}{18} \cdot 196\pi = \frac{1,078\pi}{9} \approx 376.29$.

Curved edge = $\frac{11}{18} \cdot 28\pi = \frac{154\pi}{9} \approx 53.76$.

Area $\approx 376.3 \text{ ft}^2$; edge $\approx 53.8 \text{ ft}$

Problem 18 Bicycle Wheel

(a) $s = r\theta = 13 \cdot 2.5 = 32.5 \text{ in}$.

(b) $\theta = s \div r = \frac{100}{13} \approx 7.69 \text{ radians}$, about 1.22 turns.

(a) 32.5 in (b) ≈ 7.69 rad

Problem 19 Minute Hand

35 minutes is $\frac{35}{60}$ of a turn: 210° .

Tip distance = $\frac{210}{360} \cdot 2\pi(9) = \frac{7}{12} \cdot 18\pi = 10.5\pi \approx 32.99$.

Area = $\frac{7}{12} \cdot 81\pi = 47.25\pi \approx 148.44$.

210° ; ≈ 33 in; ≈ 148.4 in²

Problem 20 Pie Chart Slice

Angle = $0.35 \cdot 360^\circ = 126^\circ$.

Area = $0.35 \cdot \pi(25) = 8.75\pi \approx 27.49$. Curved edge = $0.35 \cdot 10\pi = 3.5\pi \approx 11.00$.

126° ; ≈ 27.49 cm²; ≈ 11.00 cm